15-8. Normal Modes

A string with both ends fixed, length L, for standing wave to exist:

$$L = n\frac{\lambda}{2} \qquad n=1, 2, 3, \ldots$$

Standing wave wavelength

$$\lambda_n = \frac{2L}{n}$$

Fundamental frequency

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Harmonics, or overtones if $n>1$

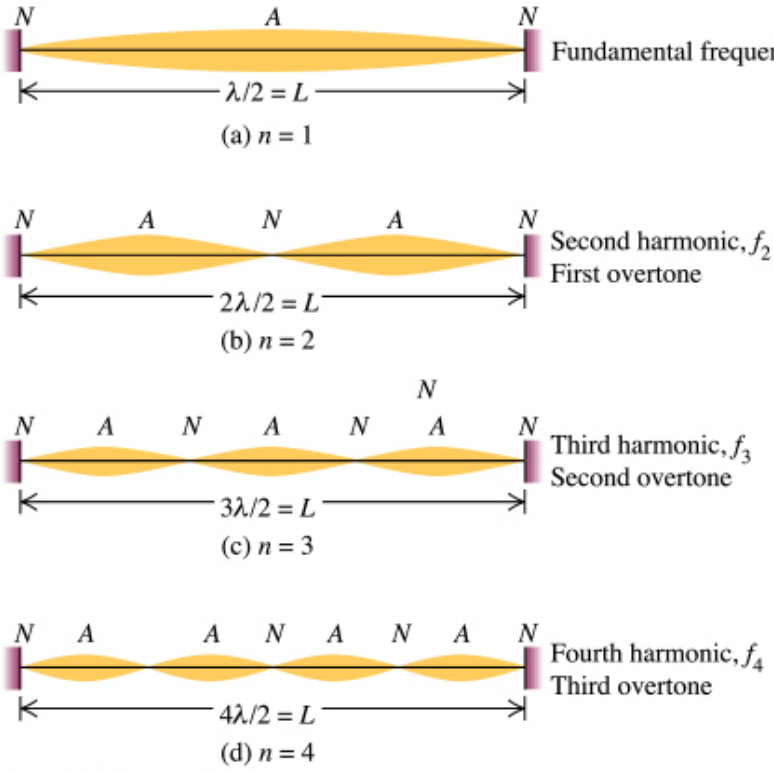
$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1$$

n^{th} harmonic is $(n-1)^{th}$ overtone

Wave function

$$y_n(x,t) = A_{SW} \sin k_n x \sin \omega_n t$$

Harmonics



Since $v = \sqrt{F/\mu}$

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

for string fixed at both ends

Normal mode: a motion in which all particles of the system move sinusoidally with the same frequency.

Resonant frequency: frequency at which standing waves are produced

Multiple resonant frequencies / normal modes for a string

Single resonant frequency for a spring / pendulam