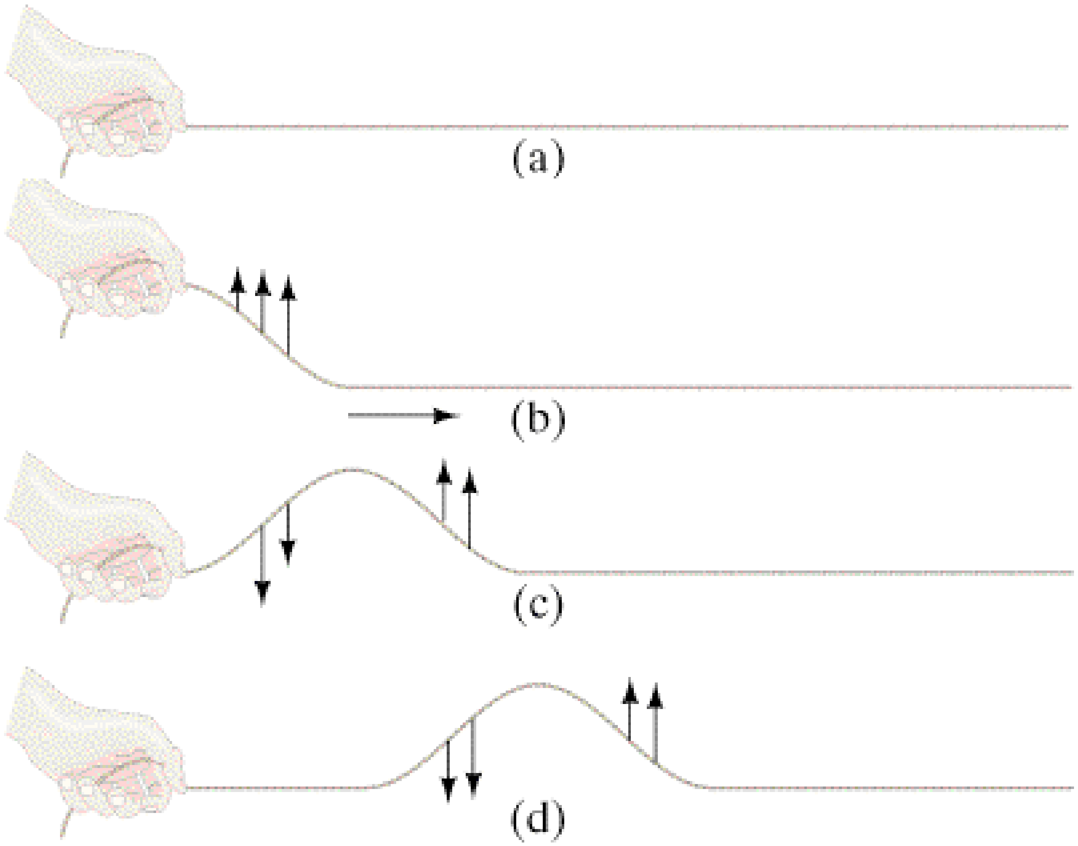
Ch15. Mechanical Waves

15-1. Introduction

Wave pulse



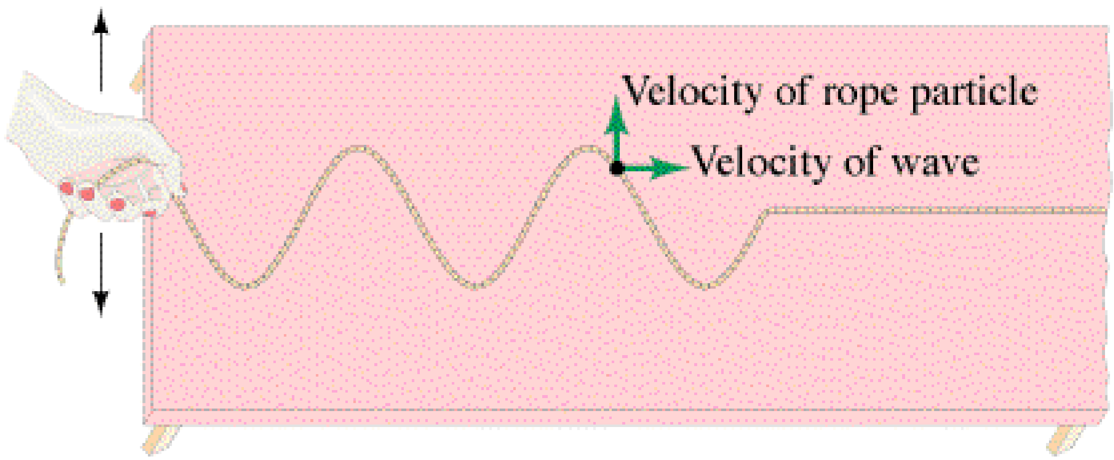
Source: disturbance + cohesive force between adjacent pieces

A wave is a disturbance that propagates through space

Mechanical wave: needs a medium to propagate

Distinctions

Wave velocity vs. particle velocity



Wave can travel & Medium has only limited motion

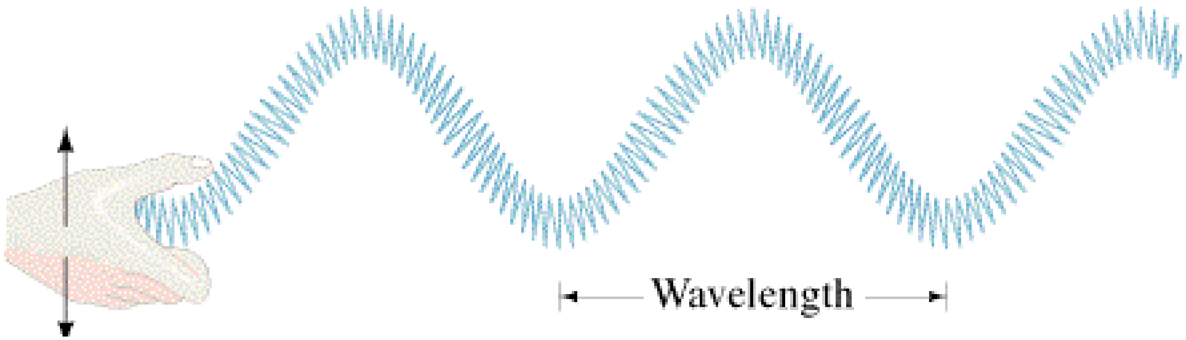
Waves are moving oscillations not carrying matter along

What do they carry/transport?

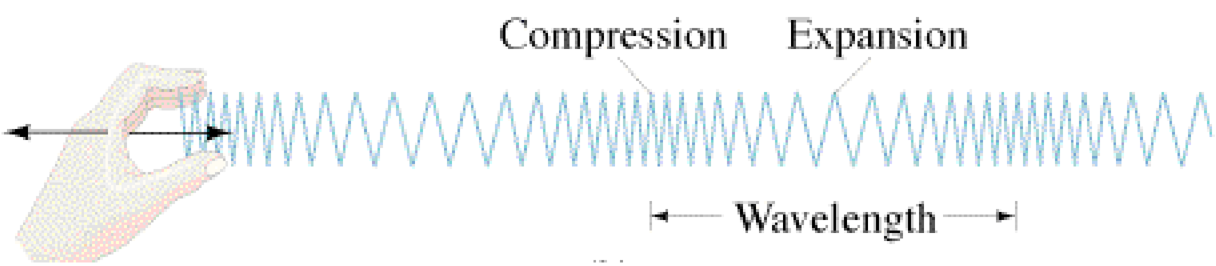
Disturbance & Energy

Types of Waves

Transverse wave



Longitudinal wave

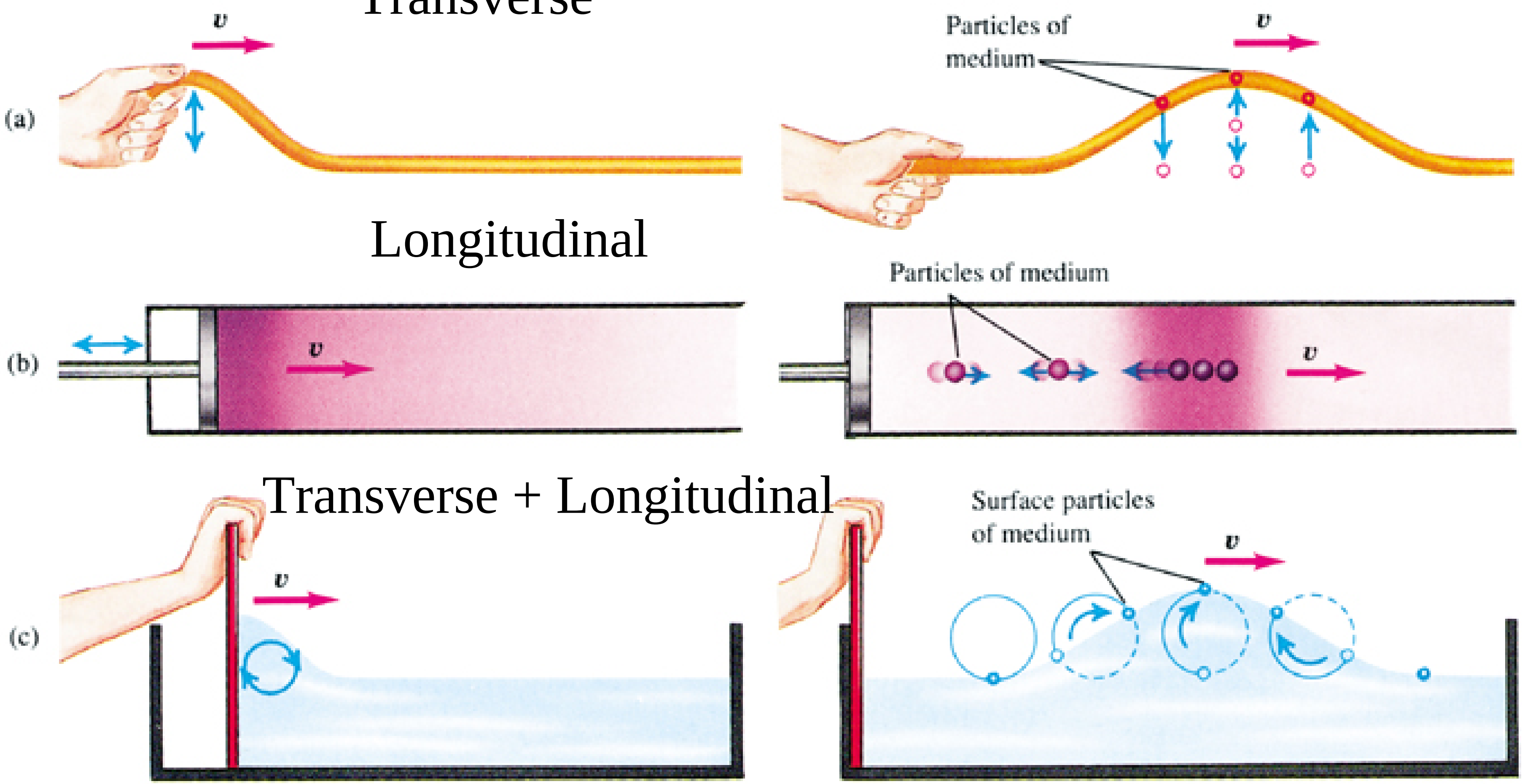


More Examples

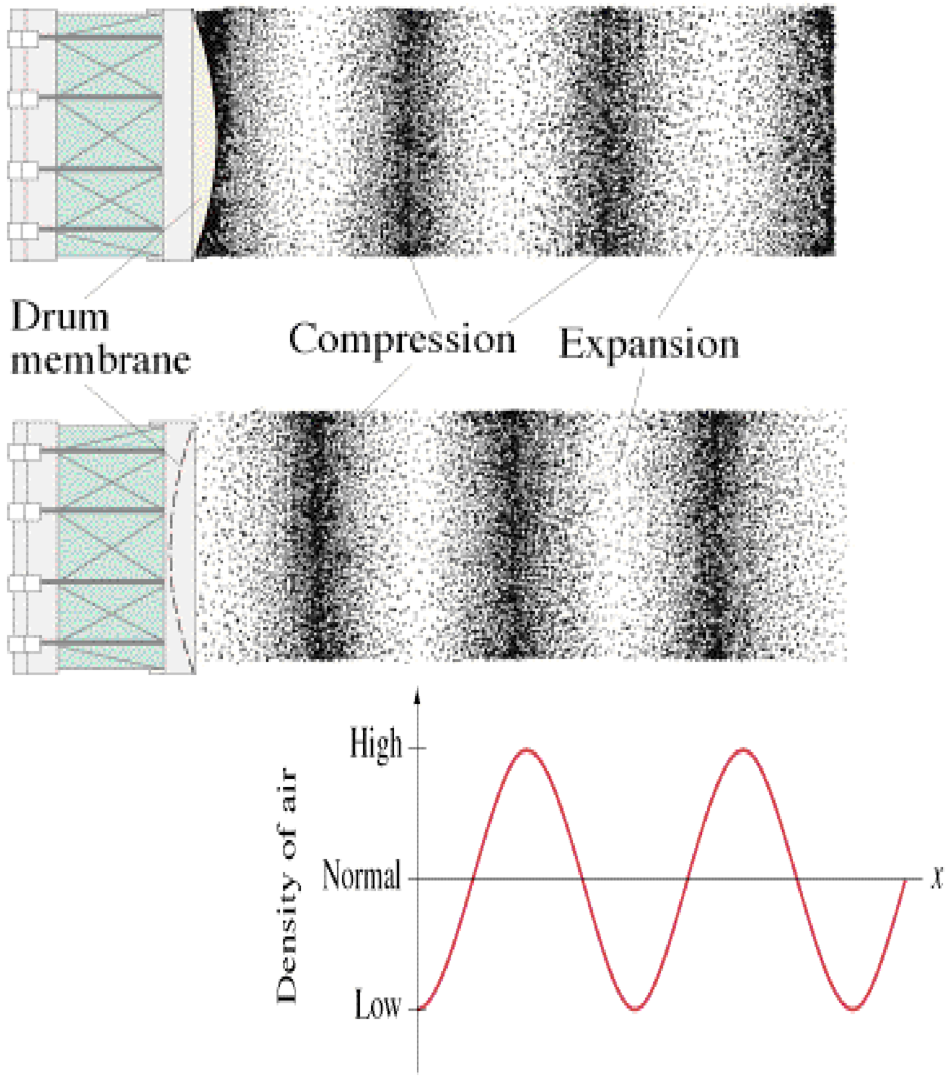
Transverse

Longitudinal

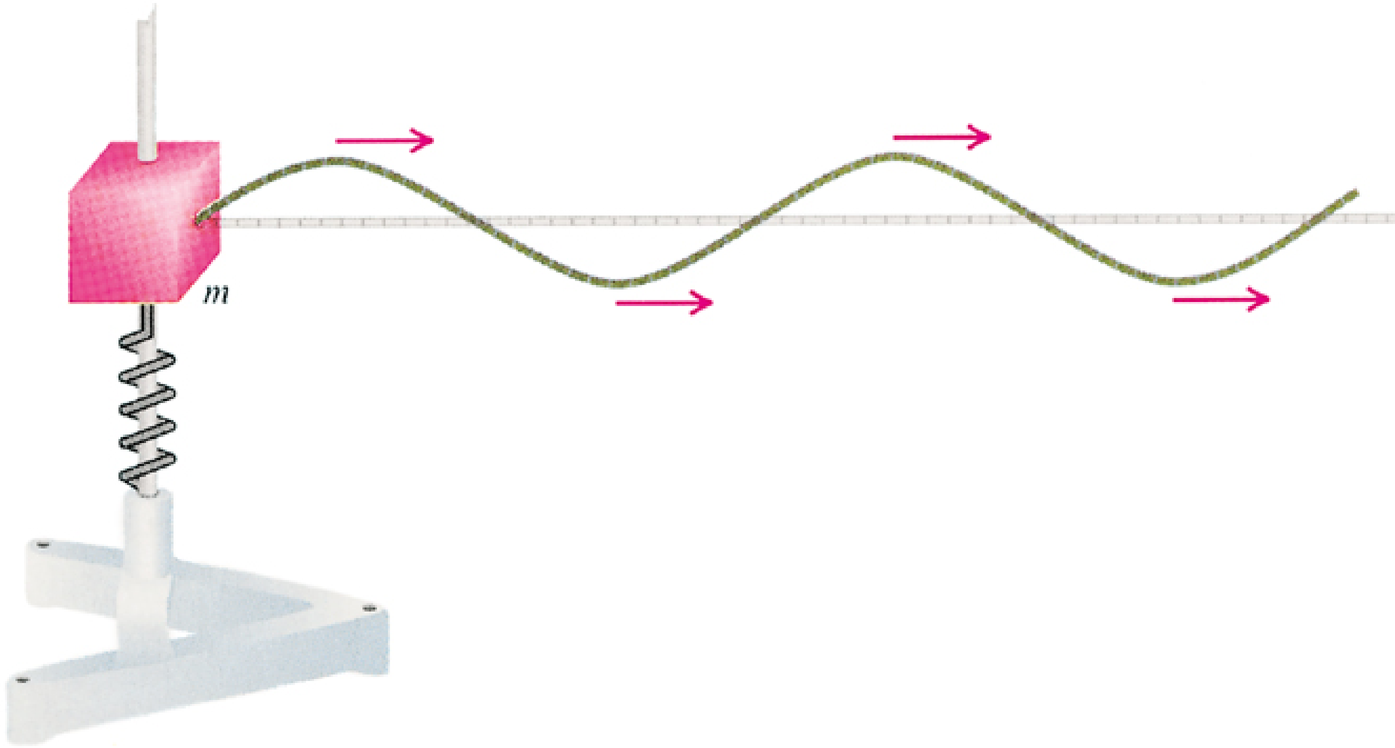
Transverse + Longitudinal



Sound Wave: Longitudinal



15-2. Periodic Waves

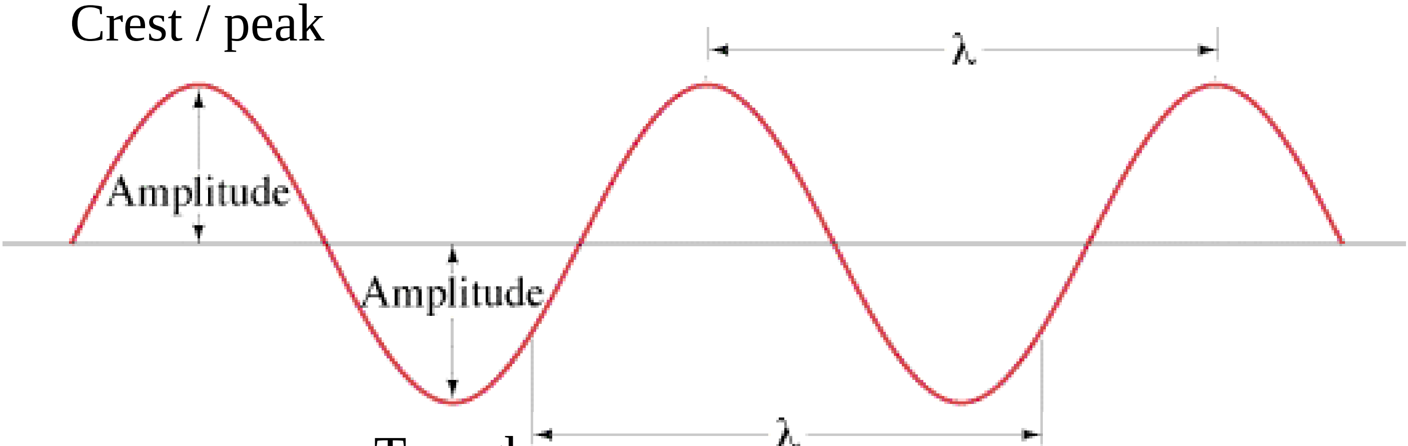


Continuous / Periodic Wave

Caused by continuous/periodic disturbance: oscillations

Characteristics of a single-frequency continuous wave

Crest / peak



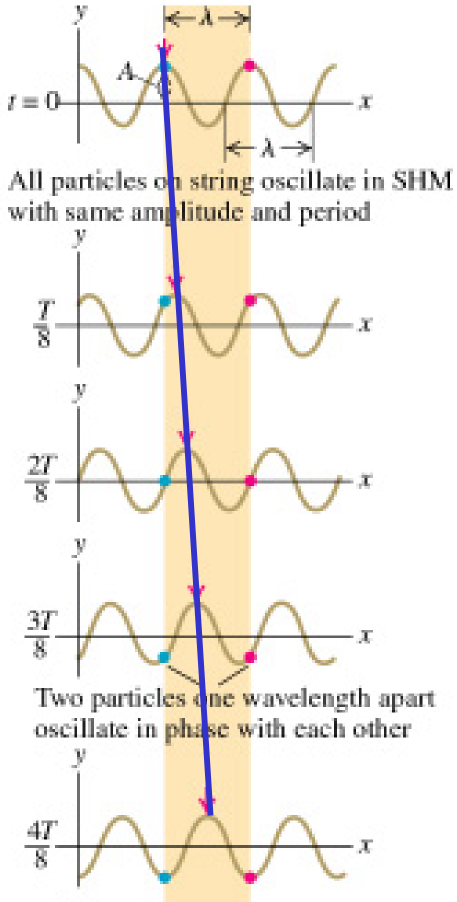
Trough

Wavelength: distance between two successive crests
or any two successive identical points on the wave

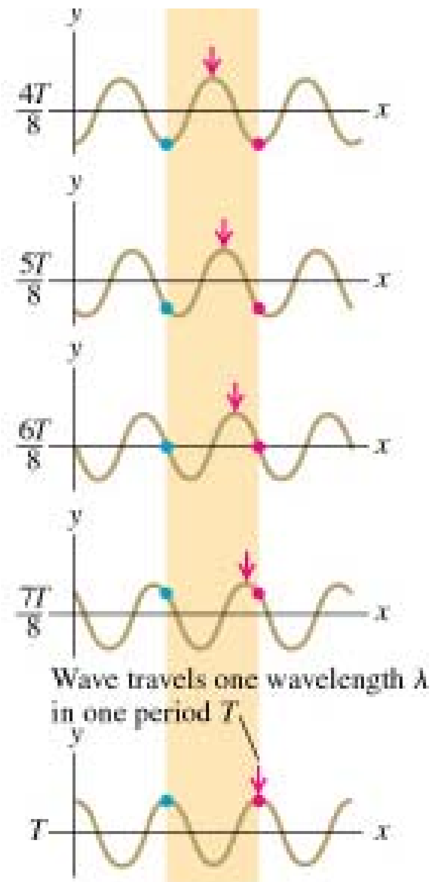
Frequency f: # of complete cycles that pass a given point per unit time

Period T: $1/f$

Wave Velocity



$$v = \lambda / T \quad = \lambda f$$



Different from particle velocity

Depends on the medium in which the wave travels

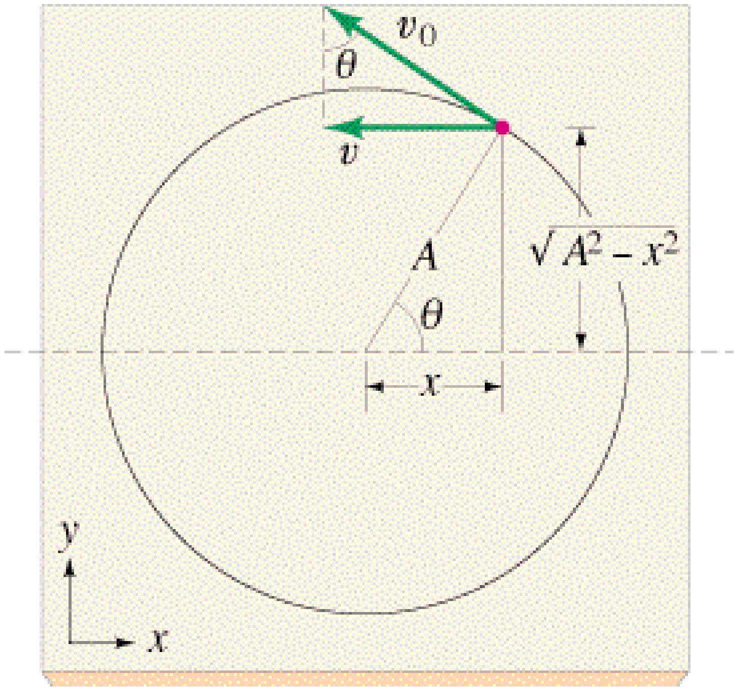
15-3. Mathematical Description

Approach:

extrapolate motion of a single point to all points

from displacement, derive velocity, acceleration, energy…

Recall SHM: Position of Oscillator

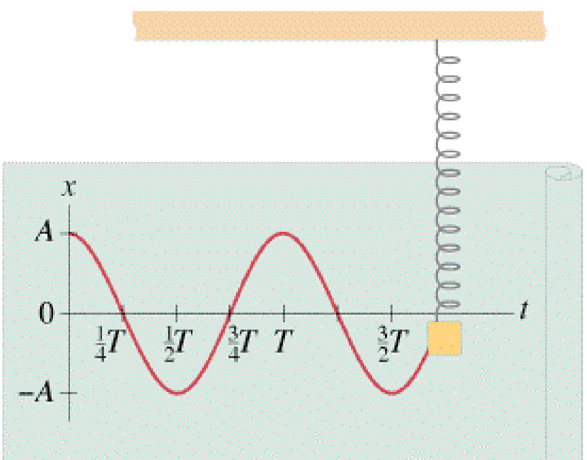


$\cos\theta = x/A$ $\theta = \omega t$

ω - angular frequency (radians / s)

$= 2\pi/T$

$= 2\pi f$



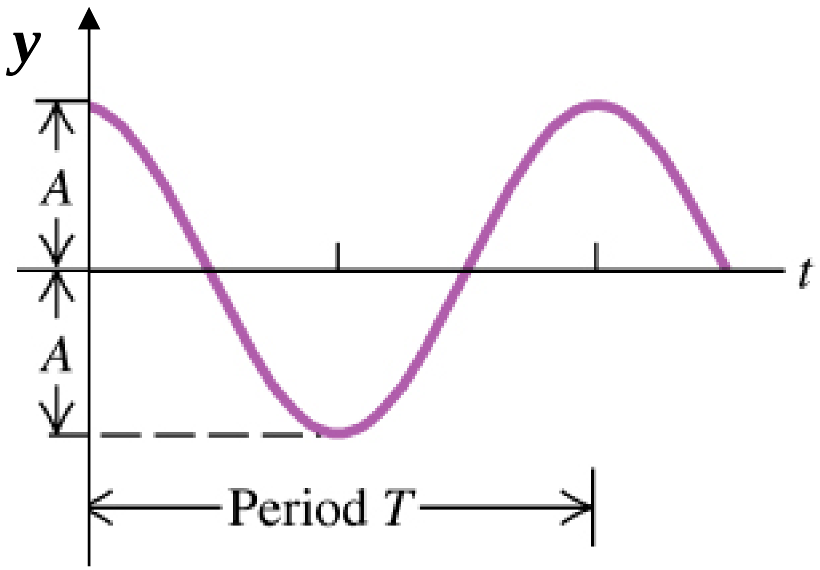
$x = A\cos\theta$

$= A\cos\omega t$

$= A\cos(2\pi t/T)$

Motion of One Point Over Time

Rewriting *y* as the particle displacement:



$$y = A\cos \omega t$$
$$= A\cos(2\pi t/T)$$

Motion of Any Point at Any Time: Wave Function

For wave moving in +x direction

$$y(x,t) = A\cos\omega(t - \frac{x}{v})$$ —— Motion at *x* trails *x*=0 by a time of *x/v*

$$y(x,t) = A\cos 2\pi f\,(t - \frac{x}{v}) = A\cos 2\pi(\frac{t}{T} - \frac{x}{\lambda}) = A\cos 2\pi(\frac{x}{\lambda} - \frac{t}{T})$$

Wave number $$k = \frac{2\pi}{\lambda}$$

$$\omega = vk$$

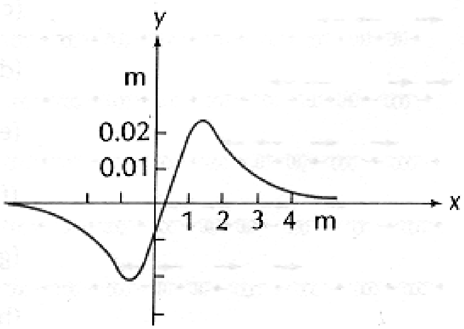
$$y(x,t) = A\cos(kx - \omega t)$$

For wave moving in -x direction

$$y(x,t) = A\cos(kx + \omega t)$$

Phase: $kx \pm \omega t$

Example



EXAMPLE 1. Suppose that at an initial time $t = 0$, the shape of a wave pulse on a string is represented by the wavefunction

$$y = f(x) = \frac{0.03x}{1 + x^4} \qquad \text{at initial time } t = 0$$

where y and x are in meters. Suppose that this wave pulse has a velocity $v = 2$ m/s toward the positive x direction. What function represents the wave pulse at time t? Plot this function when $t = 1$ s.