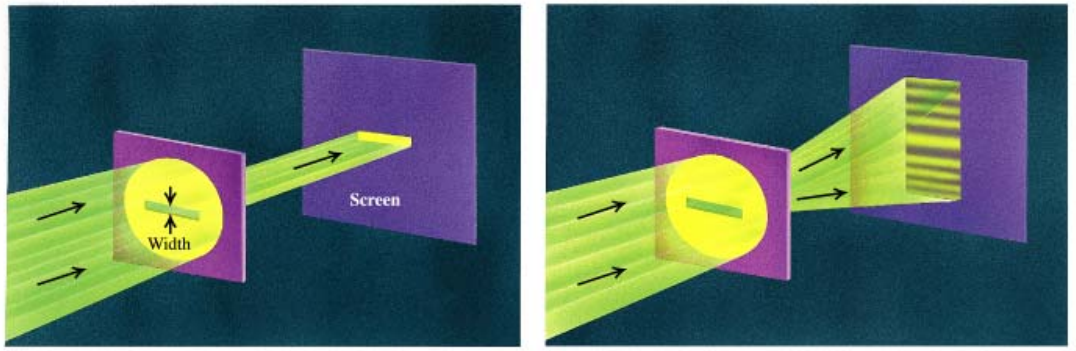
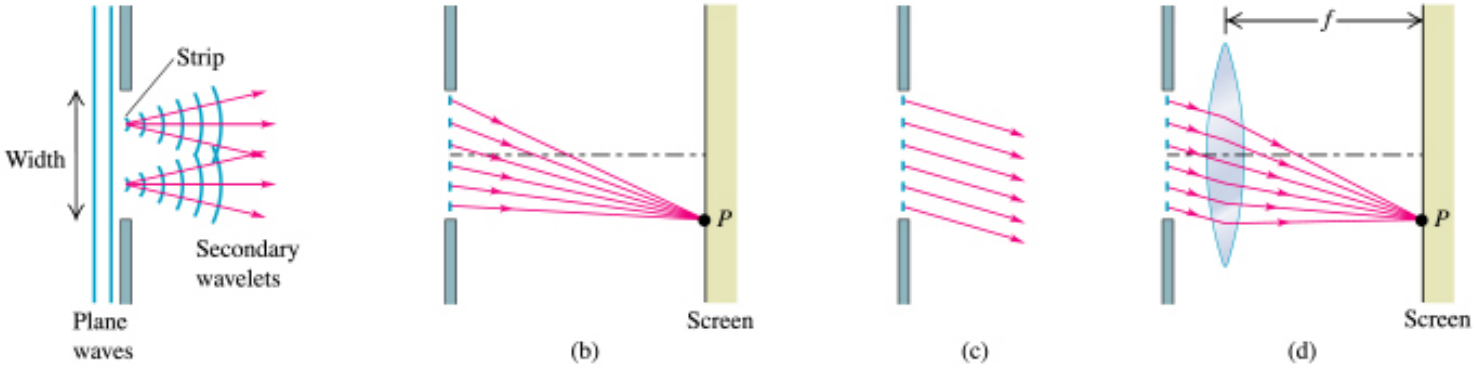
Ch 36. Diffraction

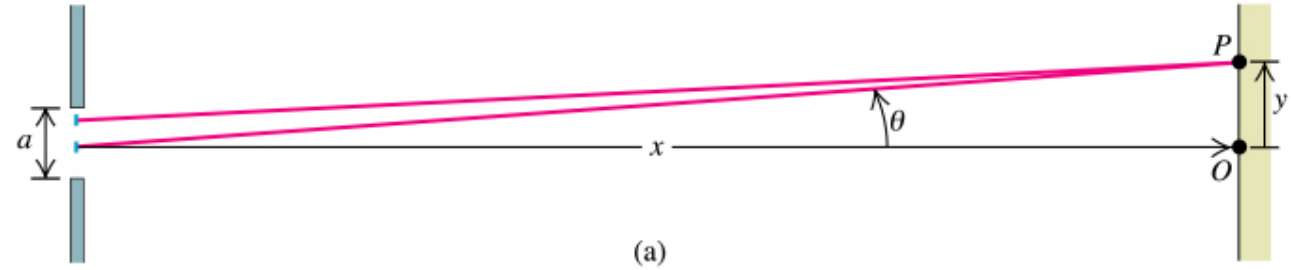
36-2. Diffraction from a Single Slit

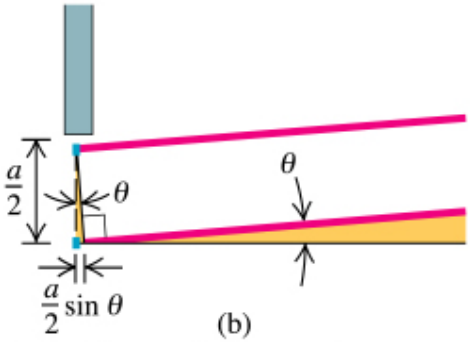
Please read 36-1 on your own.





Single Slit





Dark fringes

$\sin\theta = m\lambda/a$ $\quad m = \pm1, \pm2, \pm3\ldots$

Diffraction minima

$y_{min} = xm\lambda/a$ $\quad y_m << x$

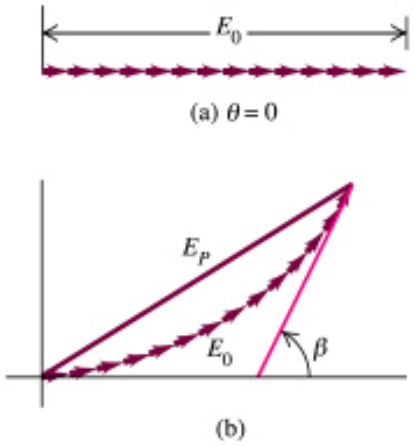
But m=0 is bright

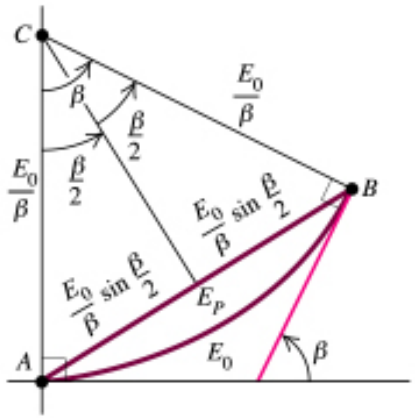
Recall double slit case

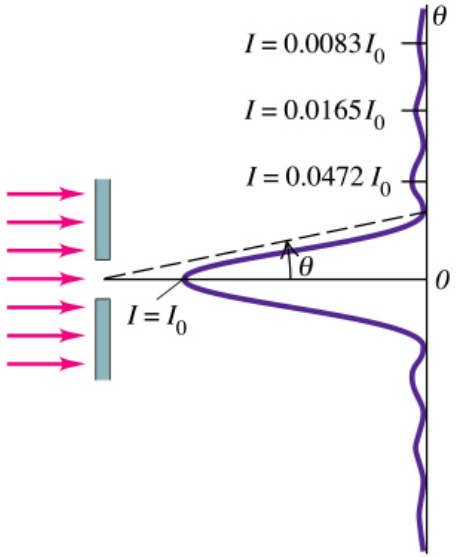
bright fringes/interference maxima: $y_{max} = Rm\lambda/d$

dark fringes/interference minima: $y_{min} = R(m + 1/2)\lambda/d$

36-3. Intensity in the Single-Slit Pattern







$$I = I_o \left[\frac{\sin(\beta/2)}{(\beta/2)} \right]^2$$

Phase difference from top and bottom of the slit:

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

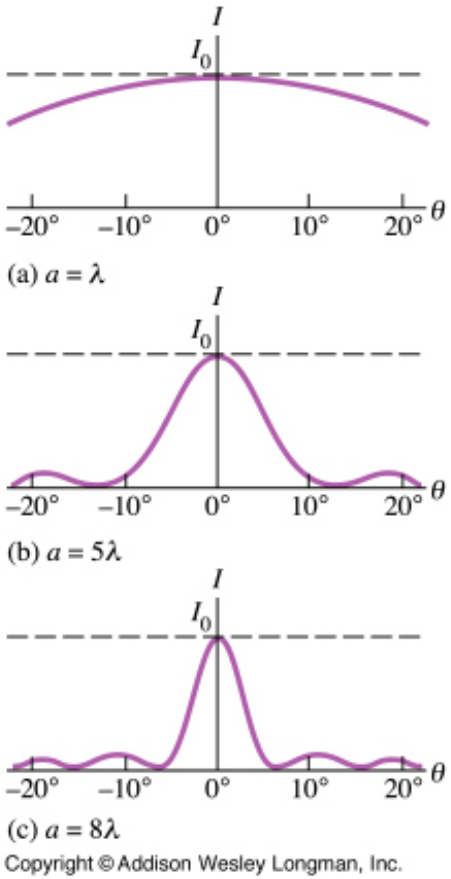
$$I = I_o \left[\frac{\sin(\pi a \sin\theta/\lambda)}{(\pi a \sin\theta/\lambda)} \right]^2$$

Intensity maxima

$$\beta \approx \pm(2m+1)\pi, (\mathrm{m} = 0, 1, 2, ...)$$

$$I_m \approx \frac{I_o}{(m+\frac{1}{2})^2 \pi^2}$$

Width of the Single-Slit Pattern



Minima: I=0

$$\sin(\pi a \sin\theta / \lambda) = 0$$

$$\pi a \sin\theta_m / \lambda = m\pi$$

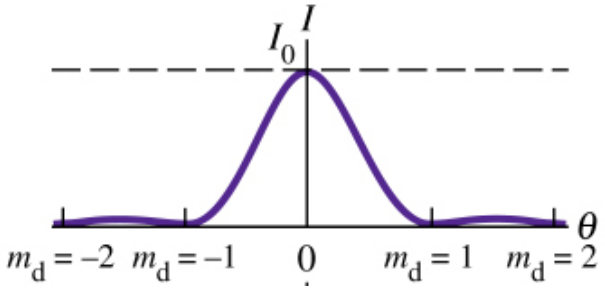
$$\boxed{\theta_m = m\lambda / a}$$

θ_1 of first minimum:

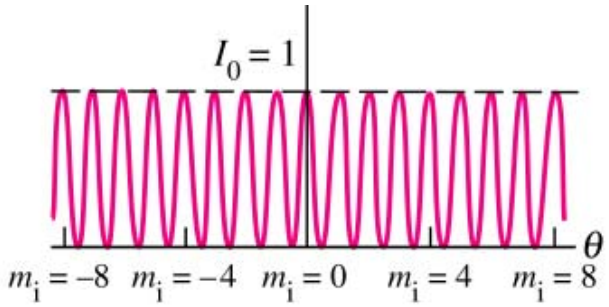
$\theta_1 = \lambda/a$

Smaller a: larger θ_1

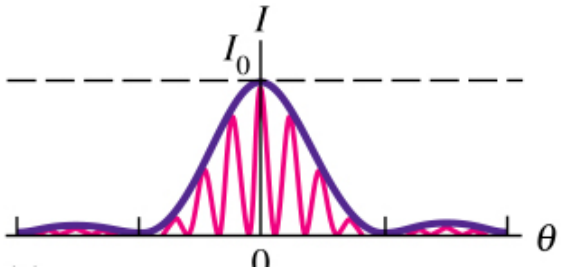
36-4. Multiple Slits



Single slit, finite width *a*



Double-slit, separation *d*,
each slit is very narrow

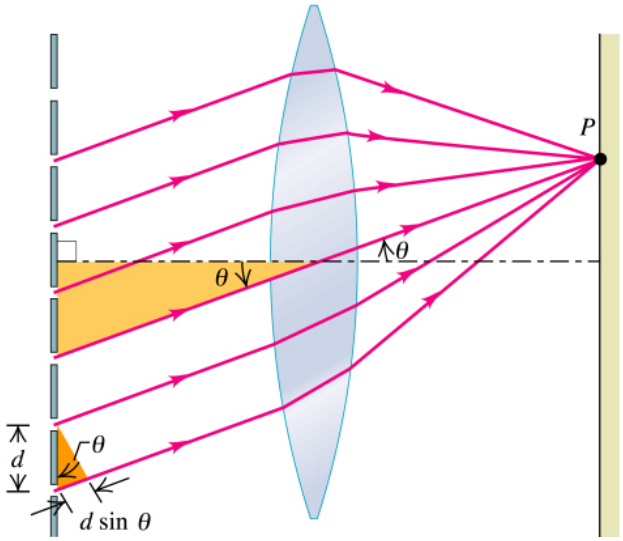


Double-slit, separation *d*,
each slit has finite width *a*

$$I = I_o \cos^2 \frac{\phi}{2} \left[\frac{\sin(\beta/2)}{(\beta/2)} \right]^2$$

$$\phi = \frac{2\pi}{\lambda} d \sin\theta \qquad \beta = \frac{2\pi}{\lambda} a \sin\theta$$

N-Slits



Similar to double-slit:

maxima at $d\sin\theta = m\lambda$

Principal maximum: ~N^2

N-1 minima between maxima