PHY 110 C
HW#3

3a $$\frac{\partial V}{\partial t} = \frac{p_0\omega}{4\pi\varepsilon_0 r}\left[-\frac{\omega}{c}\sin\omega(t-r/c)+\frac{1}{r}\cos\omega(t-r/c)\right]\cos\theta \qquad (1)$$

$$\hat{z} = \cos\theta\,\hat{r}-\sin\theta\,\hat{\theta}$$

$$\nabla\cdot\vec{A} = \nabla\cdot-\frac{\mu_0 p_0\omega}{4\pi}\ \frac{1}{r}\sin\omega\left(t-r/c\right)\left(\cos\theta\,\hat{r}-\sin\theta\,\hat{\theta}\right)$$

$$= \mu_0\varepsilon_0\left\{\frac{p_0\omega}{4\pi\varepsilon_0}\left[\frac{1}{r^2}\sin\omega\left(t-r/c\right)+\frac{\omega}{rc}\cos\omega\left(t-r/c\right)\right]\cos\theta\right\} \qquad (2)$$

$$\therefore\quad (2) = -\mu_0\varepsilon_0\times(1)$$

3b Check out $V, \vec{A}, \vec{E}, \vec{B}$ and $\langle\vec{S}\rangle$ using

$$\vec{p}_0\times\hat{r} = p_0\sin\theta\,\hat{\phi}$$

$$\hat{r}\times\left(\vec{p}_0\times\hat{r}\right) = p_0\sin\theta\left(\hat{r}\times\hat{\phi}\right) = -p_0\sin\theta\,\hat{\theta}$$

3c $$F = ma = \frac{v^2}{r} \qquad F = \frac{q^2}{4\pi\varepsilon_0\ r^2} \qquad \Rightarrow \qquad v = \left[\frac{1}{4\pi\varepsilon_0}\ \frac{q^2}{mr}\right]^{\frac{1}{2}}$$

$r_0 = 0.5\text{Å}$, $\boxed{\frac{v}{c} = 0.0075}$ safely nonrelativistic

$$P = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{q^2}{c^3}\left(\frac{v^2}{r}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{q^2}{c^3}\left(\frac{1}{4\pi\varepsilon_0}\ \frac{q^2}{mr^2}\right)^2$$

$$E = T+V = \frac{1}{2}mv^2-\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r} = \frac{1}{8\pi\varepsilon_0}\frac{q^2}{r}$$

$$P = -\frac{dE}{dt} = -\frac{1}{8\pi\varepsilon_0}\frac{q^2}{r^2}\frac{dr}{dt} = \frac{1}{4\pi\varepsilon_0}\frac{2}{3}\frac{q^2}{c}\left(\frac{1}{4\pi\varepsilon_0}\frac{q^2}{mr^2}\right)^2$$

$$\Rightarrow\quad \frac{dr}{dt} = -\frac{1}{3c}\left(\frac{q^2}{2\pi\varepsilon_0 mc}\right)^2\frac{1}{r^2} \quad\Rightarrow\quad \boxed{t = c\left(\frac{2\pi\varepsilon_0\,mc}{q^2}\right)^2 r_0^{\,3}}$$

$$\boxed{t = 1.3\times10^{-11}\ \text{secs}}$$

3d $\theta_{max} : \frac{d}{d\theta}\quad \frac{\sin^2\theta}{(1-\beta\cos\theta)^2} = 0 \qquad \Rightarrow \qquad \theta_{max} = \cos^{-1}\left[\frac{\sqrt{1+15\beta^2}-1}{3\beta}\right]$

For $v \approx c, \beta = 1-\varepsilon \quad \Rightarrow \quad \frac{\sqrt{1+15\beta^2}-1}{3\beta} \quad = \quad \frac{\sqrt{1+15(1-\varepsilon)^2}-1}{3(1-\varepsilon)} \approx 1-\frac{\varepsilon}{4}$ using Taylor expansion

$\cos\theta_{max} \approx 1-\frac{\theta^2{}_{max}}{2} \Rightarrow \theta_{max} \approx \sqrt{(1-\beta)/2}$

Let $R \equiv \frac{(dP/d\Omega\,|_{\theta max})ur}{(dP/d\Omega\,|_{\theta max})rest} = \frac{\sin^2\theta_m}{(1-\beta\cos\theta_m)^5} \quad \cdots \quad = \left(\frac{4}{5}\right)^5 \frac{1}{2\varepsilon^4}$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \Rightarrow \quad \varepsilon = \frac{1}{2\gamma^2} \quad \Rightarrow \quad \boxed{R = \frac{1}{4}\left(\frac{8}{5}\right)^5 \gamma^8}$

3e total power $P = \int \frac{dp}{d\Omega}\, d\Omega$

$$= \frac{2}{4\pi\varepsilon_0}\frac{q^2a^2}{4\pi c^3}\pi\int_0^\pi \frac{\sin\theta\, d\theta}{(1-\beta\cos\theta)^3}$$ as was done in class

change variable: $\xi = 1-\beta\cos\theta$ to do the integral

$$P = \frac{1}{4\pi\varepsilon_0} \quad \frac{q^2a^2}{4c^3} \quad \frac{1}{\beta^3} \quad \frac{8}{3} \quad \frac{\beta^3}{(1-\beta^2)^2} = \boxed{\frac{q^2}{4\pi\varepsilon_0} \quad \frac{2}{3} \quad \frac{a^2}{c^3} \quad \gamma^4}$$

Check Lienard:

$$a^2 - \left(\frac{\vec{v}}{c}\times\vec{a}\right)^2 = a^2\left[1-\left(\frac{va}{c}\,\hat{y}\right)^2\right] = a^2\left[1-\frac{v^2}{c^2}\right] = (1-\beta^2)\,a^2 = \frac{a^2}{\gamma^2}$$

$\Rightarrow \quad P = \frac{1}{4\pi\varepsilon_0} \quad \frac{2}{3} \quad \frac{q^2}{c^3} \quad \gamma^4 a^2$ Lienard