PHY 110 C
HW #5

1 Let d_a = distance to earth from point a, d_b = distance from point b.

Let t'_a = time light leaves a, t'_b = time leaves b.

Light arrives at earth $t_a = t'_a + \frac{d_a}{c}$ and $t_b = t'_b + \frac{d_b}{c}$.

Let $\Delta t = t_b - t_a$ $\quad\quad \Delta t' \equiv t'_b - t'_a$

Then $\Delta t = \Delta t' + \frac{d_b - d_a}{c}$

$= \Delta t' - \frac{v\Delta t' \cos\theta}{c} = \Delta t'\left[1 - \frac{v}{c}\cos\theta\right]$

Also $\Delta s = v\Delta t' \sin\theta$. The apparent velocity $\boxed{u = \frac{v\sin\theta}{1 - v/c\cos\theta}}$

$u > c$ if $\frac{v/c\sin\theta}{1 - v/c\cos\theta} > 1$ or $\frac{v}{c} > \frac{1}{(\sin\theta + \cos\theta)}$, e.g. at $\theta = 45^\circ$ $u > c$ if $\frac{v}{c} > \frac{1}{\sqrt{2}}$

2 Lifetime in the lab $\Delta t = \gamma\tau$. $(\tau - \text{rest lifetime})$

For velocity v, $800\text{m} \equiv d = v\Delta t = v\gamma\tau$

or $v = \frac{d}{\tau}\sqrt{1 - v^2/c^2}$. Solve: $\frac{v^2}{c^2} = \frac{1}{1 + (\tau c/d)^2}$

$v = 4/5\,c$

3 $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{5}{4}$

(a) $\gamma t = \frac{5}{4}\text{hr} = 1\text{hr}\ 15\,\text{min}$

(b) The rocket is now $\left(\frac{3}{5}c\right)\frac{5}{4}$ hr = $\frac{3}{4}c$ hr away, light will take $\frac{3}{4}$ hr. to arrive

Total time $\tau = \frac{5}{4}\text{hr} + \frac{3}{4}\text{hr} = 2\,\text{hr}$

(c) $\frac{\tau}{\gamma} = \frac{8}{5}\text{hr}$

4 Length of mast: ℓ Vertical component: $\ell \sin\theta$

Horizontal component: $\ell \cos\theta$ (on boat) $\rightarrow$ $\frac{1}{\gamma}\ell \cos\theta$ (on shore)

$$\therefore \tan\theta' = \frac{\ell \sin\theta}{1/\gamma \ell \cos\theta} = \gamma \tan\theta$$

5 trivial algebra

6 a. $dy' = dy$, but $dt' = \gamma\left(dt - v/c^2\,dx\right)$

$$\frac{dy'}{dt'} = \frac{dy}{\gamma\left(dt - v/c^2\,dx\right)} = u_y{}' = \frac{u_y}{\gamma\left(1 - v/c^2\,u_x\right)}$$

b. on shore: $\tan\theta' = -\frac{u_y{}'}{u_x{}'} = \frac{-u_y/\gamma\left(1 - v/c^2\,u_x\right)}{\left(u_x - v\right)/\left(1 - v/c^2\,u_x\right)} = \frac{1}{\gamma}\,\frac{\left(-u_y\right)}{u_x - v}$

components on boat: $u_x = -\cos\theta$; $u_y = c\sin\theta$

$$\therefore \tan\theta' = \frac{1}{\gamma}\left[\frac{\sin\theta}{\cos\theta + v/c}\right]$$