PHY 110 C

HW 6

1

a. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ -\beta & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

b. $\begin{pmatrix} \gamma & 0 & -\beta\gamma & 0 \\ 0 & 1 & 0 & 0 \\ -\beta\gamma & 0 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

c. $$\Lambda = \begin{pmatrix} \bar{\gamma} & 0 & -\bar{\beta}\bar{\gamma} & 0 \\ 0 & 1 & 0 & 0 \\ -\bar{\beta}\bar{\gamma} & 0 & \bar{\gamma} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \gamma\bar{\gamma} & -\bar{\beta}\bar{\gamma}\gamma & -\bar{\beta}\bar{\gamma} & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ -\bar{\beta}\bar{\gamma}\gamma & \beta\gamma\bar{\beta}\bar{\gamma} & \bar{\gamma} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\neq \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{\gamma} & 0 & -\bar{\beta}\bar{\gamma} & 0 \\ 0 & 1 & 0 & 0 \\ -\bar{\beta}\bar{\gamma} & \bar{\gamma} & \bar{\gamma} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2

a. identity: $1 - \tanh^2\theta \equiv \dfrac{1}{\cosh^2\theta}$

$$\Lambda = \begin{bmatrix} \cosh\theta & -\sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. 2nd identity: $\dfrac{\tanh\theta_1 - \tanh\theta_2}{1 - \tanh\theta_1 \tanh\theta_2} = \tanh(\theta_1 - \theta_2)$ (note: $v_i = c\tanh\theta_i$ for <u>any</u> v_i.)

$$\frac{u'}{c} = \frac{u/c - v/c}{1 - \frac{u}{c}\frac{v}{c}} \rightarrow \tanh\theta' = \tanh(\theta_1 - \theta_2) \Rightarrow \theta' = \theta_1 - \theta_2$$

3

a. $(12.40) \rightarrow \eta^2 = \frac{1}{1-u^2/c^2} u^2 \rightarrow \vec{u} = \frac{1}{1+\eta^2/c^2} \vec{\eta}$

b. use identity from 19a: $u = c \tanh\theta \Rightarrow \frac{1}{\sqrt{1-u^2/c^2}} = \cosh\theta \Rightarrow \eta = c\sinh\theta$

4 With $p_{tot} = \sum_i p_i$ and $E_{tot} = \sum_i E_i$

Boost p_{tot} & E_{tot} to frame where $p_{tot}' = \gamma(p_{tot} - \beta E_{tot}/c)$ is 0 $\Rightarrow p_{tot} = \frac{\beta E_{tot}}{c} = \frac{v E_{tot}}{c^2} \Rightarrow v = c^2 \frac{p_{tot}}{E_{tot}}$

5 $p^2c^2 = E^2 - m^2c^4 = (2mc^2)^2 - m^2c^4 = 3m^2c^4 \rightarrow p = \sqrt{3}mc$

& $E = 3mc^2$ (tot). conservation of E & p:

$M^2c^4 = (3mc^2)^2 - 3m^2c^4 \rightarrow \boxed{M = \sqrt{6}\,m}$ ~ 2.5m

$v = \frac{pc^2}{E} = \boxed{\frac{c}{\sqrt{3}}}$

6 Boost to the frame of <u>one</u> particle. The other:

$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \beta p\right)$ $E = \gamma mc^2 \rightarrow \gamma = \frac{E}{mc^2} \cdot p = \gamma m(-v)$ (note sign)

$E' = \gamma(E + \beta^2 \gamma mc^2) = \frac{E^2}{mc^2} + \left[\left(\frac{E}{mc^2}\right)^2 - 1\right] mc^2$ $[\text{with } \beta^2\gamma^2 = \gamma^2 - 1]$

$= \frac{2E^2}{mc^2} - mc^2$

For beam $E = 30$ GeV, $E' = 2 \cdot 900 - 1 =$ 1800 GeV = 60 E.