PHY 110 C
HW 7

12.52 Differentiate the Maxwell's equations $\partial_\mu \partial_\upsilon F^{\mu\upsilon} = \mu_0 \partial_\mu J^\mu$

but $\partial_\mu \partial_\upsilon = \partial_\upsilon \partial_\mu$ and $F^{\mu\upsilon} = -F^{\upsilon\mu}$

therefore $\partial_\mu \partial_\upsilon F^{\mu\upsilon} = -\partial_\upsilon \partial_\mu F^{\upsilon\mu} = -\partial_\mu \partial_\upsilon F^{\mu\upsilon} = 0$ quantity equal to its own negative

$\therefore \quad \partial_\mu J^\mu = 0$

12.53 $\partial_\upsilon G^{\mu\upsilon} = 0$ gives $\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

We will show that $\partial_\lambda F_{\mu\upsilon} + \partial_\upsilon F_{\upsilon\lambda} + \partial_\upsilon F_{\lambda\mu} = 0$ also give them.

There are only two more trivial cases: (1) μ, ν, λ are permutations of 1, 2, 3 and (2) only one of μ, ν, λ is 0.

Example of (1): $\mu = 1$, $\nu = 2$, $\lambda = 3$

$$\partial_3 F_{12} + \partial_1 F_{23} + \partial_2 F_{31} = 0 \quad \text{gives} \quad \frac{\partial B_z}{\partial z} + \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \quad \text{i.e.} \quad \nabla \cdot \vec{B} = 0$$

Example of (2): $\mu_0 = 0$, $\nu = 1$, $\lambda = 3$

$$\partial_2 F_{01} + \partial_0 F_{12} + \partial_1 F_{20} = 0 \quad \text{gives} \quad \frac{\partial}{\partial y}\left(-\frac{E_x}{c}\right) + \frac{\partial}{\partial (ct)}(B_z) + \frac{\partial}{\partial x}\left(\frac{E_y}{c}\right) = 0$$

$$\text{i.e.} \quad -\frac{\partial B_z}{\partial t} + \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) = 0 \quad \text{or} \quad -\frac{\partial B_z}{\partial t} = \left(\nabla \times \vec{E}\right)_z$$

12.55 Consider boost in x-direction

$$\left(\partial^\circ \phi\right)' = -\frac{\partial \phi}{\partial (ct')} = -\frac{\partial \phi}{\partial (ct)}\frac{\partial (ct)}{\partial (ct')} - \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial (ct')} = -\frac{\partial \phi}{\partial (ct)}\gamma - \frac{\partial \phi}{\partial x}\beta\gamma = \gamma(\partial^\circ \phi) - \beta\gamma(\partial^1 \phi)$$

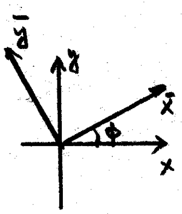
$$\left(\partial^1 \phi\right) = \frac{\partial \phi}{\partial x'} = \frac{\partial \phi}{\partial (ct)}\frac{\partial (ct)}{\partial x'} + \frac{\partial \phi}{\partial x}\frac{\partial x}{\partial x'} = \frac{\partial \phi}{\partial (ct)}\beta\gamma + \frac{\partial \phi}{\partial x}\gamma = -\beta\gamma\,(\partial^\circ \phi) + \gamma\,(\partial^1 \phi)$$

They transform like contravarient vectors.

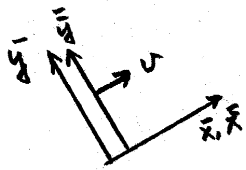
12.56 $F_{\mu\upsilon} = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu$

$$\partial_\lambda F_{\mu\upsilon} + \partial_\mu F_{\upsilon\lambda} + \partial_\upsilon F_{\lambda\mu} = \partial_\lambda \partial_\upsilon A_\mu - \partial_\lambda \partial_\mu A_\upsilon + \partial_\mu \partial_\lambda A_\upsilon - \partial_\mu \partial_\upsilon A_\lambda + \partial_\upsilon \partial_\mu A_\lambda - \partial_\upsilon \partial_\lambda A^\mu = 0$$

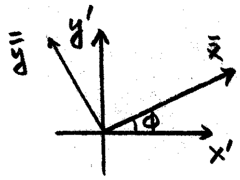
From 12.53 therefore $\partial_\upsilon G^{\mu\upsilon} = 0$

12.57 

$$\begin{pmatrix} c\bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \Lambda_1 \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$



$$\begin{pmatrix} c\bar{\bar{t}} \\ \bar{\bar{x}} \\ \bar{\bar{y}} \\ \bar{\bar{z}} \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \Lambda_2 \begin{pmatrix} c\bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}$$

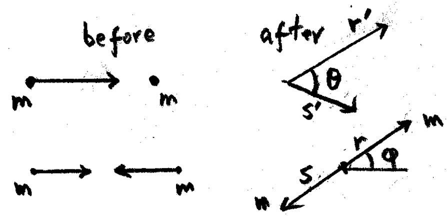


$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c\bar{\bar{t}} \\ \bar{\bar{x}} \\ \bar{\bar{y}} \\ \bar{\bar{z}} \end{pmatrix} = \Lambda_3 \begin{pmatrix} c\bar{\bar{t}} \\ \bar{\bar{x}} \\ \bar{\bar{y}} \\ \bar{\bar{z}} \end{pmatrix}$$

$$\Lambda = \Lambda_3\Lambda_2\Lambda_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

12.59 LAB:

CM:



In LAB with for the 4-momentum, we know

$\cos\theta = \dfrac{\vec{r}' \cdot \vec{s}'}{|\vec{r}'||\vec{s}'|}$. In the CM frame, calling the magnitude of the 3-momentum, "p",

we know $r^\mu = (E/c, p\cos\phi, p\sin\phi, 0)$ no prime

$s^\mu = (E/c, -p\cos\phi, -p\sin\phi, 0)$

Now boost these to the primed (LAB) frame using

$$E = \gamma mc^2\,; \quad p = -\gamma mv\,; \qquad \beta = \frac{-\,pc}{E}$$

$$r'_x = \lambda\left(p\cos\phi + \frac{pc}{E}\frac{E}{c}\right) = \gamma p\,(1+\cos\phi) \qquad r'_y = p\sin\phi$$

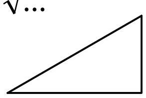
$$s'_x = \gamma p\,(1-\cos\phi) \qquad\qquad s'_y = -p\sin\phi$$

sub all this in $\cos\theta$ expression, above:

$$\cos\theta = \frac{\gamma^2 p^2\,(1-\cos^2\phi) - p^2\sin^2\phi}{\sqrt{[\gamma^2 p^2 (1+\cos\phi)^2 + p^2\sin^2\phi\,][\gamma^2 p^2 (1-\cos\phi)^2 + p^2\sin^2\phi]}}$$

$$= \frac{\sin\phi}{\sqrt{\dfrac{4}{(\gamma^2-1)^2} + \dfrac{4}{\gamma^2-1} + \sin^2\phi}} \quad = \quad \frac{1}{\sqrt{\dfrac{4\gamma^2}{(\gamma^2-1)^2}\cdot\dfrac{1}{\sin^2\phi} + 1}}$$

Trig: $\sqrt{\ldots}$



$$\frac{2\gamma}{\gamma^2-1}\cdot\frac{1}{\sin^2\phi}$$

1

$$\Rightarrow \quad \tan\theta = \frac{2\gamma}{\gamma^2-1}\cdot\frac{1}{\sin^2\phi}$$

But $\gamma^2 - 1 = \gamma^2\left(1-\dfrac{1}{\gamma^2}\right) = \gamma^2\left(1-1+\dfrac{v^2}{c^2}\right) = \gamma^2\,\dfrac{v^2}{c^2}$

$$\boxed{\tan\theta = \frac{2c^2}{\gamma v^2 \sin\phi}}$$