PHY 110C
HW 8

12.42 a) $\vec{E}_0 = \frac{\sigma_0}{\sqrt{2}\varepsilon_0}\left(-\hat{x}+\hat{y}\right)$

b) $E_y = \gamma E_{y0} \quad \Rightarrow \quad \vec{E} = \frac{\sigma_0}{\sqrt{2}\varepsilon_0}\left(-\hat{x}+\gamma\hat{y}\right)$

c) Prob 12.10 $\Rightarrow \quad \tan\theta = \gamma$

d) $\tan\theta_E = \frac{E_y}{E_x} = -\gamma \qquad$ i.e. $\theta_E = -\theta$

12.45 Frame C: $\boxed{\vec{E}_c = \frac{-q\hat{j}}{4\pi\varepsilon_0 d^2}} \qquad \boxed{\vec{B}_c = 0} \qquad \boxed{\vec{F}_c = \frac{-q^2\hat{y}}{4\pi\varepsilon_0}}$

Frame A: Technically, you're <u>BACK</u> transforming. You know $\vec{E}$ in C which is moving w.r.t. you.

$\vec{E}_A = \gamma\vec{E}_c = \boxed{\frac{-\gamma q\hat{y}}{4\pi\varepsilon_0 d^2}} \quad \cdot B_{AZ} = +\frac{v}{c^2}E_{Ay} \cdot \boxed{\frac{-\gamma q\hat{z}}{4\pi\varepsilon_0 d^2}\frac{v}{c^2} = \vec{B}_A}$

$\boxed{\vec{F}_A} = q\vec{E}_A + \left(-v\hat{x}\right)\times\vec{B}_A = \ldots = \boxed{\frac{-\gamma q^2}{4\pi\varepsilon_0 d^2}\left(1+\frac{v^2}{c^2}\right)\hat{y}}$

Frame B: Same trick but A → B:

$\boxed{\vec{E}_B} = \gamma\left(E_{Ay} + vB_{Ay}\right) = \ldots = \boxed{\frac{-q\gamma^2}{4\pi\varepsilon_0 d^2}\left(1+\frac{v^2}{c^2}\right)\hat{y}}$

$\boxed{\vec{B}_B = \frac{-\gamma^2 q}{4\pi\varepsilon_0 d^2}\frac{2v}{c^2}\hat{z}} \qquad \boxed{\vec{F}_B} = q\vec{E}_B = \boxed{\frac{-\gamma^2 q^2}{4\pi\varepsilon_0 d^2}\left(1+\frac{v^2}{c^2}\right)\hat{y}}$

12.46 a) $\vec{E}'\cdot\vec{B}' = E'_x B'_x + E'_y B'_y + E'_z B'_z = E_x B_x + \gamma\left(E_y - vB_z\right)\gamma\left(B_y + \frac{v}{c^2}E_z\right) + \ldots$

$= \ldots = E_x B_x + E_y B_y + E_z B_z = \vec{E}\cdot\vec{B}$

b) $E'^2 - c^2 B'^2 = \left[E_x{}^2 + \gamma^2\left(E_y - vB_z\right)^2 + \ldots\right] - c^2\left[B_x{}^2 + \gamma^2\left(By + \frac{v}{c^2}E_z\right)^2\right] + \ldots$

$= \ldots = \left(E_x{}^2 + E_y{}^2 + E_z{}^2\right) - c^2\left(B_x{}^2 + B_y{}^2 + B_z{}^2\right) = E^2 - c^2 B^2$

c) If $\vec{B} = 0 \Rightarrow \quad E^2 - c^2B^2 = E^2$
If $\vec{E}' = 0 \quad \Rightarrow \quad E'^2 - c^2 B'^2 = -c^2 B'^2$
These two can't be equal as they have different signs.