

## Physics 115A

Spring 2006

### Problem Set 1

Due by 4:00 pm Friday, 4/14 (in class, at my office, or in my mailbox)

Look up any integrals you need for these problems.

#### 1. Gaussian wave functions (30 points)

Consider the wave function

$$\psi(x) = Ke^{ax-bx^2}$$

where  $K$ ,  $a$ , and  $b$  are constants.

- Use the condition of normalization to find  $K$ .
- Find the expectation values  $\langle x \rangle$  and  $\langle x^2 \rangle$ .
- Find the expectation values  $\langle p \rangle$  and  $\langle p^2 \rangle$ .
- Find the standard deviations (“uncertainties”)  $\Delta x$  and  $\Delta p$ .
- Show that the Heisenberg uncertainty relations hold.
- Find the probability current  $J(x)$ .

#### 2. Probabilities (20 points)

Consider a wave function

$$\psi(x) = \begin{cases} 0 & x < -1 \\ A(-1 + x^2) & -1 < x < 1 \\ 0 & x > 1 \end{cases}$$

(You might want to sketch  $\psi$  and  $|\psi|^2$  to help visualize the problem.)

- Find the constant  $A$  from the normalization requirement.
- Let  $b$  be an arbitrary number between  $-1$  and  $1$ . What is the probability that the particle is located in the range  $-1 < x < b$ ?
- If you answered question b correctly, you should find that the probability is one when  $b = 1$ . Explain why.