Physics 115A **Spring 2006**

Corrected Version, Problem Set 2
Due in class Wednesday, 4/26

1. **Momentum space** (30 points)
We have seen that a particle with definite momentum p has a wave function of the form

$$\psi_p(x) = C\, e^{ipx/\hbar}.$$

If the momentum is not definite, the wave function is a *superposition* of wave functions with definite momenta, that is a sum of wave functions $\psi_p(x)$. The most general form of such a superposition is an integral (which is, after all, basically an infinite sum):

$$\psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int \phi(p,t) e^{ipx/\hbar} dp$$

where $\phi(p,t)$ is a nearly arbitrary function. You may recognize this as a Fourier transform.

a. For such a wave function $\psi(x,t)$, find $\hat{p}\psi(x,t)$, where $\hat{p}$ is the momentum operator. (Your answer should be an integral expression.)

b. Find the expectation value $\langle \hat{p} \rangle$ as a single integral over p. Hints:
   i. You will need both ψ^* and ψ, each of which is an integral. Don't mix up the two integration variables!
   ii. Use the fact that
   $$\frac{1}{2\pi\hbar} \int e^{iax/\hbar} dx = \delta(a)$$
   where $\delta(a)$ is the Dirac delta function.

c. Find the expectation value $\langle \hat{p}^n \rangle$ of an arbitrary power of the momentum operator as a single integral over p. (Once you have done part b, this should be easy.)

d. Let $F(p)$ be any function of momentum that can be expanded in a power series (that is, $F(p) = a_0 + a_1 p + a_2 p^2 + \ldots$). Using part c, find the expectation value $\langle F(\hat{p}) \rangle$ as a single integral over p.

2. **Square well superposition** (20 points)
Note: this is almost Problem 2.5 of Griffiths, and requires section 2.2. This is the corrected version (oops!)—the wave function should be a superposition of ψ_1 and ψ_4 (not ψ_3).

A particle in an infinite square well has as its initial wave function an even mixture of the first and third stationary states:

$$\psi(x,0) = A[\psi_1(x) + \psi_4(x)]$$

where $\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$.

a. Normalize $\psi(x,0)$, that is, find A.
b. Find $\psi(x,t)$ and $|\psi(x,t)|^2$. Express the latter in terms of sines or cosines.
c. Compute $\langle x \rangle$. Note that it oscillates in time. What are the angular frequency and amplitude of the oscillation?
d. Compute $\langle \hat{p} \rangle$.
e. Compute the expectation value of the Hamiltonian $\hat{H}$. Compare it with E_1 and E_2.
f. A *classical* particle in this well would bounce back and forth between the walls. If its energy is equal to the expectation value you found in (e), what is the frequency of the classical motion? How does it compare with the quantum frequency you found in (c)?