Physics 115A Spring 2006

Problem Set 3
Due by 4:00 pm Wednesday, 5/3 (in class, at my office, or in my mailbox)

Look up any integrals you need for these problems.

1. **Commutators** (15 points)
   a. Show that for any operators $\hat{A}$, $\hat{B}$, and $\hat{C}$,

$$[\hat{A}+\hat{B},\hat{C}] = [\hat{A},\hat{C}] + [\hat{B},\hat{C}]$$

   and that

$$[\hat{A}\hat{B},\hat{C}] = [\hat{A},\hat{C}]\hat{B} + \hat{A}[\hat{B},\hat{C}]$$

   b. Find the commutators $[x^2, \hat{p}^2]$ and $[x\hat{p}, \hat{p}x]$.
   c. Let $f(x)$ be an arbitrary function of x. Find $[\hat{p}, f]$.

2. **Harmonic oscillator exercises** (15 points)
Consider a harmonic oscillator in the nth energy state ψ_n.
   a. Find the uncertainties Δx and Δp and the product $\Delta x \Delta p$.
   b. Find the expectation values $\langle \hat{T} \rangle$, $\langle \hat{V} \rangle$, and $\langle \hat{H} \rangle$, where $\hat{T} = \frac{\hat{p}^2}{2m}$, $\hat{V} = \frac{1}{2}m\omega^2 x^2$, and $\hat{H} = \hat{T} + \hat{V}$.
   c. Find the expectation values $\langle x\hat{p} \rangle$, $\langle \hat{p}x \rangle$, and $\langle x\hat{p} - \hat{p}x \rangle$.

3. **Perturbation theory and the harmonic oscillator** (20 points)
This problem has a long introduction followed by a calculation for you to do.

Often it is impossible to find exact solutions of the time-independent Scrödinger equation $\hat{H}\psi = E\psi$. In 115B, you will learn an approximation technique called stationary state perturbation theory. Here is a short summary of the results:

Suppose we can write the Hamiltonian as $\hat{H} = \hat{H}_0 + \lambda \hat{W}$, where $\hat{H}_0$ is a simpler Hamiltonian whose Schrödinger equation we *can* solve and λ is small (so the full Hamiltonian is a small "perturbation" from one we understand). Let the time-independent wave functions for $\hat{H}_0$ be denoted $\psi_n^{(0)}$, with energies $E_n^{(0)}$ — that is, $\hat{H}_0 \psi_n^{(0)} = E_n^{(0)} \psi_n^{(0)}$. Let

$$W_{mn} = \int \psi_m^{(0)*} \hat{W} \psi_n^{(0)}\, dx$$

Then the energy levels of $\hat{H}$ are

$$E_n = E_n^{(0)} + \lambda W_{nn} + \lambda^2 \sum_{m \neq n} \frac{|W_{mn}|^2}{E_n^{(0)} - E_m^{(0)}} + \ldots$$

where the omitted terms are of order λ^3 or higher, and therefore very small.

Now the calculation: consider a harmonic oscillator $H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 x^2$ with a perturbation $\hat{W} = x$. Find the energy levels E_n to order λ^2.