

**Problem Set 6**

**Due by 4:00 pm Wednesday, 5/31 (in class, at my office, or in my mailbox)**

**1. Simultaneous eigenvectors: an example (10 points)**

Consider the two matrices

$$M_1 = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}, \quad M_2 = \begin{pmatrix} d & e \\ f & g \end{pmatrix} \quad \text{with } a \neq c$$

- a. Find the eigenvectors and eigenvalues of  $M_1$ . (There are two different eigenvectors; be sure you find both.)
- b. Suppose that both of the eigenvectors you found in part a are *also* eigenvectors of  $M_2$ . Find what restrictions this places on  $M_2$ . Show that for  $M_1$  and  $M_2$  to have two simultaneous eigenvectors, the commutator  $[M_1, M_2]$  must be zero.

**2. Simultaneous eigenvectors in general (10 points)**

Let  $\hat{A}$  and  $\hat{B}$  be hermitian operators. A simultaneous eigenstate is a state obeying

$$\hat{A}|\psi_n\rangle = a_n|\psi_n\rangle, \quad \hat{B}|\psi_n\rangle = b_n|\psi_n\rangle$$

Show that if  $\hat{A}$  and  $\hat{B}$  do not commute, they cannot have a *complete* set of simultaneous eigenstates. (Hint: look at  $[A, B]|\psi_n\rangle$  for any particular simultaneous eigenstate, and then think about what this would imply for a complete set.)

**3. Expectation values (10 points)**

Let  $\hat{A}$  be a hermitian operator, with a complete set of eigenstates  $|a_n\rangle$ . The expectation value of  $\hat{A}$  in a state  $|\psi\rangle$  is defined to be

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$$

Show that

$$\langle \hat{A} \rangle = \sum_n a_n |\langle a_n | \psi \rangle|^2$$

**4. Virial theorem (10 points)**

Show that

$$\frac{d}{dt} \langle \hat{x}\hat{p} \rangle = 2\langle \hat{T} \rangle - \left\langle x \frac{dV}{dx} \right\rangle$$

where  $T$  and  $V$  are the kinetic and potential energies. In a stationary state, the left-hand side is zero: explain why. The resulting relation between  $T$  and  $V$  is called the virial theorem, and is important in many fields of physics—it tells us how energy is divided up among various components of a system.

**5. Hermitian operators (10 points)**

- a. The operators  $\hat{A}$ ,  $\hat{B}$ , and  $\hat{C}$  are all hermitian, with  $[\hat{A}, \hat{B}] = \hat{C}$ . Show that  $\hat{C} = 0$ .
- b. The operator  $\hat{F}$  is defined by

$$\hat{F}\psi(x) = \psi(x+a) + \psi(x-a)$$

Determine whether  $\hat{F}$  is hermitian.