

Problem Set 7

Due by 4:00pm Wednesday, June 7 (in class, at my office, or in my mailbox)

1. **Kaon decay** (50 points)

A neutral kaon (called K^0) is an elementary particle that can be produced by a subatomic interaction called a “strong interaction.” The kaon is unstable, and decays radioactively into other particles by a different type of subatomic interaction, called a “weak interaction.” The neutral kaon, like any other elementary particle, has an antiparticle, the antikaon (called \bar{K}^0). This problem asks a series of questions about the kaon-antikaon system. Note that this is a two-state system, with a basis $\{|K^0\rangle, |\bar{K}^0\rangle\}$ that you can take to be orthonormal.

a. The “charge conjugation” operator, denoted \widehat{CP} , is an operator that changes a kaon state to an antikaon state and vice versa,

$$\widehat{CP}|K^0\rangle = |\bar{K}^0\rangle, \quad \widehat{CP}|\bar{K}^0\rangle = |K^0\rangle$$

Show that the eigenvalues of \widehat{CP} are ± 1 . (It may help to write \widehat{CP} as a matrix.)

b. The eigenstate of \widehat{CP} with eigenvalue $+1$ is called $|K_S\rangle$, while the eigenstate with eigenvalue -1 is called $|K_L\rangle$. Find these eigenstates, and write them in the $\{|K^0\rangle, |\bar{K}^0\rangle\}$ basis, that is, as linear combinations of $|K^0\rangle$ and $|\bar{K}^0\rangle$. Be sure to normalize them.

(The subscripts S and L stand for “short” and “long”: the K_S particle has a lifetime of about 10^{-10} seconds, while the K_L particle has a longer lifetime, about 5×10^{-8} seconds.)

c. Normally, energies in quantum mechanics must be real. To describe an *unstable* system, though—one that decays at a finite rate—it is useful to allow complex energies. Recall that a stationary state has a wave function that looks like

$$\Psi(x, t) = \psi(x)e^{-iEt/\hbar}$$

Suppose that E has an imaginary part, $E = E_0 - i\Gamma$. Find the total probability of finding the particle *anywhere* as a function of time. What is the half-life (the time for the probability to fall by a factor of two) in terms of Γ ?

(Γ is called the “width” or “decay width.” Because the particle has a finite lifetime, its energy satisfies an uncertainty principle, with $\Delta E \sim \Gamma$, so if one plots the measured energy one finds a curve with width of order Γ .)

d. The eigenstates $|K_S\rangle$ and $|K_L\rangle$ are observed to have different masses, and therefore different energies. They also decay differently, and have different half-lives. We can therefore write

$$E_S = m_S c^2 - i\Gamma_S, \quad E_L = m_L c^2 - i\Gamma_L$$

(where I have ignored the extra, small kinetic energy contribution to $E_{S,L}$). If the wave function at time $t = 0$ is $|\Psi(0)\rangle = |K_S\rangle$, find $|\Psi(t)\rangle$. Do the same if the initial wave function is $|K_L\rangle$.

Using the results of the preceding questions, you should now be able to answer the following:

e. Suppose a kaon $|K^0\rangle$ is produced (by a strong interaction) at time $t = 0$. What is the probability of finding a kaon $|K^0\rangle$ at a later time t ? What is the probability of finding an antikaon $|\bar{K}^0\rangle$ at time t ?