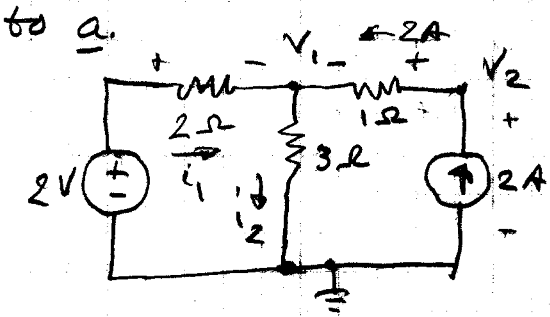
Physics 116A Exam 1 10/29/2003 Solutions

10 1. $i_2 = 12V/6\Omega = 2A$, $i_3 = 12V/3\Omega = 4A$.

KCL: $i_1 + 3i_2 = i_2 + i_3 \Rightarrow i_1 = i_3 - 2i_2 = 4A - 4A = \underline{0}$.

40 2. (a)

i) $V_{oc} \equiv V_2$ in the drawing here since $i = 0$ in the 3Ω resistor connected to $\underline{a}$.



KCL: $i_1 + 2\,A = i_2$

KVL: (left loop) $2V = 2i_1 + 3i_2$

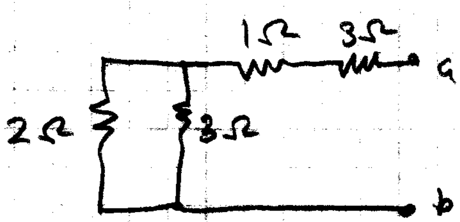
$= 2i_1 + 3i_1 + 6$

$i_1 = -4/5\ A$

KVL: (outer loop) $2V + 2\times 1\Omega = 2i_1 + V_2 = -8/5 + V_2 \Rightarrow V_2 = 5.6\ V$

$\boxed{V_{oc} = V_2 = \underline{5.6V}}$ [or can use nodal analysis to find V_1]

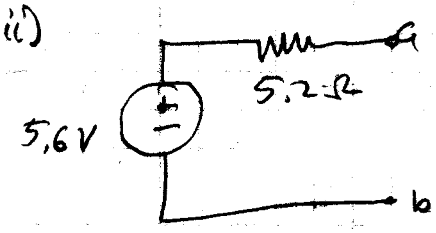
ii) Set independent sources to 0. { V → short circuit; I → open "

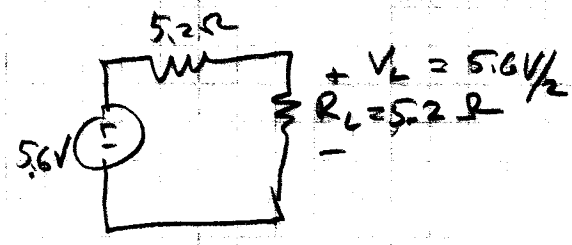


$\Rightarrow R_0 = 4\Omega + 2\Omega \| 3\Omega$

$= 4\Omega + \frac{6\Omega}{2+3} = \underline{5.2\ \Omega}$.

iii)





(b) Maximum power when $R_L = R_0 = 5.2\Omega$

(c) The resulting circuit is a voltage divider with two equal resistors so $V_L = 5.6\ V/2$

$$P = \frac{V_L^2}{R_L} = \frac{5.6^2\ V^2}{4 \times 5.2\ \Omega} = \underline{1.51\ W}.$$

30 3. $\omega = 2$ rad/s.

(a) $\underset{\sim}{Z} = 2\Omega \parallel \underset{\sim}{Z}_c \qquad \underset{\sim}{Z}_c = \frac{1}{j\omega C} = -\frac{j\Omega}{2 \times \frac{1}{4}} = \underline{\underline{-2j\,\Omega}}$

$$= \frac{\cancel{2}(-2j)\,\Omega}{\cancel{2}_1 - \cancel{2}_1 j} = \frac{-2j\,\Omega}{1-j} = \underline{\underline{(1-j)\,\Omega}}$$

(b) $\underset{\sim}{Z}$ and the 1Ω resistor ~~form~~ a voltage divider driven by V_o.

$$\underset{\sim}{V}_- = \frac{1\Omega}{1\Omega + \underset{\sim}{Z}} \underset{\sim}{V}_o = \frac{V_o}{1+(1-j)} = \frac{1}{2-j} \underset{\sim}{V}_o$$

$$= \frac{\underset{\sim}{V}_o}{2.24 \measuredangle -26.6^\circ} = 0.45 \measuredangle 26.6^\circ \underset{\sim}{V}_o$$

(c) $\underset{\sim}{V}_- = \underset{\sim}{V}_s$ (since $V_+ = V_-$)

$$\underset{\sim}{H} = \frac{\underset{\sim}{V}_o}{\underset{\sim}{V}_s} = \frac{\underset{\sim}{V}_o}{\underset{\sim}{V}_-} = \frac{1}{0.45} \measuredangle -26.6^\circ = \underline{\underline{2.23 \measuredangle -26.6^\circ}}$$

(d) $\underset{\sim}{V}_o = \underset{\sim}{H}\, \underset{\sim}{V}_s = 2.23 \measuredangle -26.6^\circ \times 6 \measuredangle 0^\circ = \underline{\underline{13.4 \measuredangle -26.6^\circ \text{ V}}}$

$v_o = 13.4 \cos(2t - 26.6^\circ)$ V.

20 4. (a) Again, we have a voltage divider.

$$\tilde{V}_o = \frac{\tilde{Z}_c \tilde{V}_s}{R + \tilde{Z}_L + \tilde{Z}_c}$$

$$\tilde{H} = \frac{\tilde{V}_o}{\tilde{V}_s} = \frac{\tilde{Z}_c}{R + \tilde{Z}_L + \tilde{Z}_c} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{1}{1 - \omega^2 LC + j\omega RC}$$

(b) $$\tilde{H} = \frac{1}{1 - \omega^2/\omega_r^2 + \frac{j\omega}{\omega_r Q}}$$ since $RC = \frac{1}{\omega_r Q}$.

(c) $$\tilde{H} = \frac{1}{1 - (\omega/\omega_r)^2 + j\sqrt{2}\,\omega/\omega_r}$$

i) $\omega \ll \omega_r \Rightarrow \tilde{H} \approx 1 \Rightarrow |H| \approx 1$.

ii) $\omega \gg \omega_r \Rightarrow \tilde{H} \approx \frac{-1}{(\omega/\omega_r)^2} \Rightarrow |H| \approx \left(\frac{\omega_r}{\omega}\right)^2$ (falls like $\frac{1}{\omega^2}$)

iii) $\omega = \omega_r \Rightarrow \tilde{H} = \frac{1}{j\sqrt{2}} \Rightarrow |H| = \frac{1}{\sqrt{2}}$ (half-power point)

iv) Low pass filter.

(d) $$\tilde{H}(\tilde{s}) = \frac{1}{1 + \frac{1}{\omega_r^2}s^2 + \frac{\sqrt{2}}{\omega_r}s}$$ where $\tilde{s} = \sigma + j\omega$

$\tilde{H}(s)$ has two poles corresponding to the two roots of the quadratic in the denominator,

$$1 + \frac{1}{\omega_r^2}s^2 + \frac{\sqrt{2}}{\omega_r}s = 0.$$

[If we define $s' = s/\omega_r$, poles at $s' = -\frac{\sqrt{2}}{2}(1 \pm j)$]