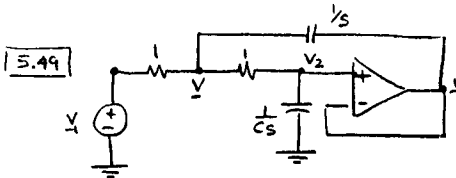
Physics 116A Fall 2004 Problem Set 5 Solutions

From Instructor's Manual for Fundamentals of Electrical Engineering, 2nd Ed.

5.49



By voltage division, $\underline{V}_2 = \frac{1/Cs}{1+1/Cs}\underline{V} = \frac{1}{Cs+1}\underline{V}$

$\therefore \underline{V} = (Cs+1)\underline{V}_2$

By KCL at node $\underline{V}_1$

$$\frac{\underline{V}-\underline{V}_1}{1} + \frac{\underline{V}-\underline{V}_2}{1} + \frac{\underline{V}-\underline{V}_2}{1/s} = 0$$

$$\underline{V} - \underline{V}_1 + \underline{V} - \underline{V}_2 + s\underline{V} - s\underline{V}_2 = 0$$

$$(s+2)\underline{V} - \underline{V}_1 - (s+1)\underline{V}_2 = 0$$

$$\rightarrow (s+2)(Cs+1)\underline{V}_2 - (s+1)\underline{V}_2 = \underline{V}_1$$

$$(Cs^2+2Cs+s+2-s-1)\underline{V}_2 = \underline{V}_1$$

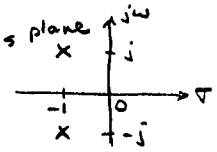
$$(Cs^2+2Cs+1)\underline{V}_2 = \underline{V}_1$$

$$\therefore \frac{\underline{V}_2}{\underline{V}_1} = \frac{1}{Cs^2+2Cs+1}$$

$$\frac{\underline{V}_2}{\underline{V}_1} = \frac{1/C}{s^2+2s+1/C}$$

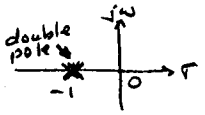
(a) $C = \frac{1}{2}F \Rightarrow \frac{\underline{V}_2}{\underline{V}_1} = \frac{2}{s^2+2s+2}$

$$= \frac{2}{(s+1+j)(s+1-j)}$$



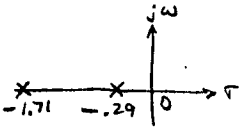
(b) $C = 1F \Rightarrow \frac{\underline{V}_2}{\underline{V}_1} = \frac{1}{s^2+2s+1}$

$$= \frac{1}{(s+1)^2}$$



(c) $C = 2F \Rightarrow \frac{\underline{V}_2}{\underline{V}_1} = \frac{1/2}{s^2+2s+1/2}$

$$= \frac{1/2}{(s+.29)(s+1.71)}$$



6.2 $L = 1\,m \quad d = 10^{-3}\,m \Rightarrow r = 0.5\times10^{-3}\,m \quad R = 2.2\times10^{-2}\,\Omega \quad i = 2\,A$

(a) $J = \frac{i}{A} = \frac{i}{\pi r^2} = \frac{2}{\pi(0.5\times10^{-3})^2} = \underline{2.55\times10^{6}\ A/m^2}$

(b) $J = nqu \Rightarrow u = \frac{J}{nq}$

$$u = \frac{2.55\times10^{6}}{(8.43\times10^{28})(1.6\times10^{-19})} = \underline{1.89\times10^{-4}\ m/s}$$

(c) $v = Ri = (2.2\times10^{-2})(2) = 4.4\times10^{-2}\ V$

$$\mathcal{E} = 4.4\times10^{-2}\ V/m$$

$$\mu = \frac{u}{\mathcal{E}} = \frac{1.89\times10^{-4}}{4.4\times10^{-2}} = \underline{4.3\times10^{-3}\ m^2/V\text{-}s}$$

(d) $\sigma = nq\mu = (8.43\times10^{28})(1.6\times10^{-19})(4.3\times10^{-3}) = \underline{5.8\times10^{7}\ \mho/m}$

6.5 Si $\quad A = 1\,mm^2 = (10^{-3})^2\,m^2 = 10^{-6}\,m^2 \quad T = 300K \quad R = 50k\Omega$

$n_i = 1.5\times10^{16}\,m^{-3} \quad \mu_n = 0.13\,m^2/V\text{-}s \quad \mu_p = 0.05\,m^2/V\text{-}s$

(a) $\sigma = (n\mu_n + p\mu_p)q = (n_i\mu_n + n_i\mu_p)q$

$$\sigma = n_i(\mu_n+\mu_p)q$$

$$\sigma = (1.5\times10^{16})(0.13+0.05)(1.6\times10^{-19}) = \underline{4.32\times10^{-4}\ \mho/m}$$

(b) $R = \frac{L}{\sigma A} \Rightarrow L = R\sigma A = 50k(4.32\times10^{-4})(10^{-6}) = 2.16\times10^{-5}\,m$

$$= \underline{21.6\,\mu m}$$