Pulse Problem Solutions - Winter 2005

1. $V_{out}(t) = V(1 - e^{-t/\tau})$ $\quad \tau = RC$.

R.T. = Rise time $= t_2 - t_1$

$0.9V = V(1 - e^{-t_2/\tau})$ $\qquad\qquad$ $0.1V = V(1 - e^{-t_1/\tau})$

$0.9 = 1 - e^{-t_2/\tau}$ $\qquad\qquad$ $0.1 = 1 - e^{-t_1/\tau}$

$e^{-t_2/\tau} = 0.1$ $\qquad\qquad$ $e^{-t_1/\tau} = 0.9$

$e^{t_2/\tau} = 10$ $\qquad\qquad$ $e^{t_1/\tau} = 10/9$

$t_2 = \tau \ln 10$ $\qquad\qquad$ $t_1 = \tau(\ln 10 - \ln 9)$

$R.T. = t_2 - t_1 = \tau \ln 9 = \underline{RC \ln 9}$ $\quad$ but $RC = \frac{1}{2\pi f_c} = \frac{1}{2\pi\, BW}$

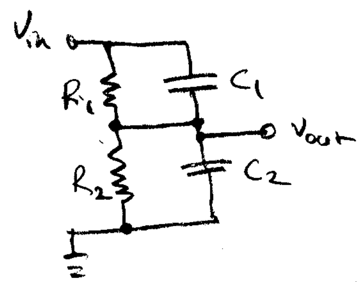
$$R.T. = \frac{\ln 9}{2\pi\, BW} = \underline{\frac{0.35}{BW}}. \qquad (\text{or } R.T. = 2.2\tau)$$

2. $BW = f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 1k\Omega \times 50pF} = 3.2\text{ MHz}.$

$R.T. = 0.35/BW = 0.35/3.2\text{ MHz} = \underline{110\text{ ns}}.$

3.

(a) $H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$




$R_1 = 9R_2$

$C_1 = C_2/9$

$R_2 = 1M\Omega$

$C_2 = 50pF$

($\underline{\text{note}}$: $R_1C_1 = R_2C_2$)

where

$$Z_1 = R_1 \parallel Z_{C_1} = \frac{R_1 \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

$$Z_2 = R_2 \parallel Z_{C_2} = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$H(j\omega) = \frac{\frac{R_2}{1 + j\omega R_2 C_2}}{\frac{R_1}{1 + j\omega R_1 C_1} + \frac{R_2}{j\omega R_2 C_2}} = \frac{R_2}{\frac{R_1(1 + j\omega R_2 C_2)}{1 + j\omega R_1 C_1} + R_2}$$

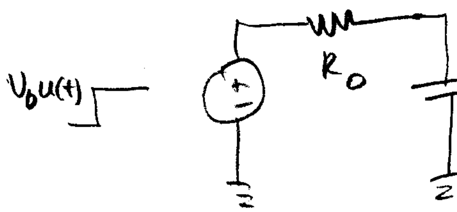
$$= \frac{R_2}{\frac{9R_2(1 + j\omega R_2 C_2)}{1 + j\omega(\cancel{9}R_2)C_2/\cancel{9}} + R_2} = \frac{R_2}{9R_2 + R_2} = \underline{0.1}.$$

3 (b). $Z_{in} = Z_1 + Z_2 = \dfrac{R_1}{1+j\omega R_1 C_1} + \dfrac{R_2}{1+j\omega R_2 C_2} = \dfrac{9R_2 + R_2}{1+j\omega R_2 C_2} = \dfrac{10 R_2}{1+j\omega R_2 C_2}$

$$= \frac{10 R_2}{1 + j\omega (10 R_2)(C_2/10)} = \frac{R'}{1+j\omega R'C'} = Z' \text{ for } R' \| Z_{c'}.$$

$R' = 10\,M\Omega$, $C' = 5\,pF$.

(c) Equiv. circuit:



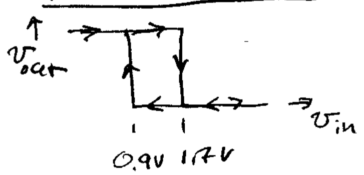

where $R_0 = R_s \| R' = 1k\Omega \| 10\,M\Omega \cong 1k\Omega$

$V_0 = \dfrac{R'}{R_s + R'} V$

$= \dfrac{10\,M\Omega}{1k\Omega + 10\,M\Omega} V \cong V.$

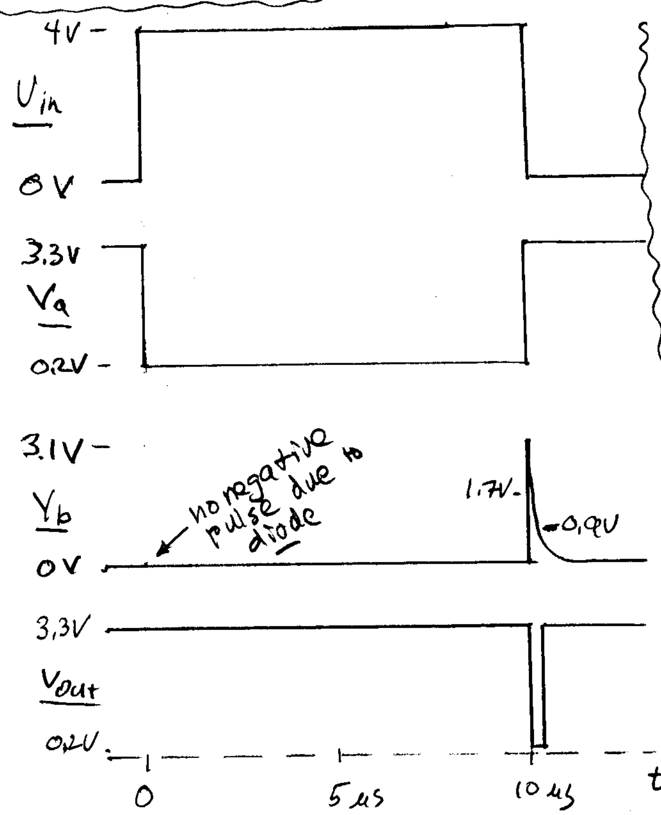
R.T. $= 2.2\tau = 2.2 \times R_0 C'$
$= 2.2 \times 1k\Omega \times 5pF = 11\,ns$. (or use BW relation)

Pulse Problem 4: Note terminology of problem is a bit unclear. 0.9V = threshold for schmitt trigger low to high transition. 1.7V = threshold for high to low.




S.T. transfer fcn.

Neglect schmitt trigger propagation delay and assume the output is a step fcn. (actual device will have delay, pulse rise time).




Timing Diagram

width of output pulse:

Consider $V_b$. At $t = 10\,\mu s$, the positive-going edge of $V_a$ leads to ($t \geq 10\,\mu s$):

$$V_b(t') = 3.1V\left(e^{-t'/\tau}\right) u(t')$$

where $t' = t - 10\,\mu s$, and

$\tau = RC = 1000\,\Omega \times 100 \times 10^{-12} F$
$= 0.10\,\mu s$

The positive transition causes $V_{out}$ to go to 3.3 V at $t = 10\,\mu s$. It falls to 0.2 V when $V_b = 0.9V$.

$0.9V = 3.1V\left(e^{-t'/0.10\,\mu s}\right)$

$t' = 0.10\,\mu s \ln(3.1/0.9) = 0.12\,\mu s$

(width of "notch" in output pulse.)

⇒ circuit produces negative-going pulse synchronized with trailing edge of input pulse.