Physics 116A
12/8/04

* Clarify signs in phase shift oscillator example

* Amplifier frequency response
  - Series, shunt capacitor corner freq.
  - Small signal models of BJT and FET showing shunt C's which limit HF response
  - Miller's Theorem
* gain - BW product
* BJT example (9.54)
* FET example (9.57)

clarify sign in phase shift oscillator:

$\boxed{AB = -1}$ <u>assumed</u> negative feedback. } condition for oscillation: "Barkhausen* Criterion"

Taking this into account, the amplifier gain $\boxed{A = R_F/R}$. (−1 there already in $\triangleright$ −1)

<u>For the RC network,</u>

$$B = \frac{V_1}{V_0} = \frac{\omega^3 R^3 C^3}{\omega RC(\omega^2 R^2 C^2 - 5) + j(1 - 6\omega^2 R^2 C^2)}$$

<u>B must be real</u> for $AB = -1$ so

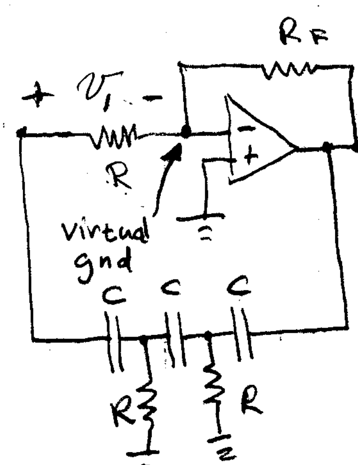
$$1 - 6\omega^2 R^2 C^2 = 0 \Rightarrow \omega = \boxed{\omega_0 = \frac{1}{\sqrt{6}\,RC}}.$$

$$B = \frac{\omega_0^2 R^2 C^2}{(\omega_0^2 R^2 C^2 - 5)}, \quad \omega_0^2 R^2 C^2 = \frac{1}{6}$$

$$= \frac{1/6}{1/6 - 5} = \frac{1}{1 - 30} = -\frac{1}{29}$$

$$AB = -1 \Rightarrow \frac{R_F}{R}\left(-\frac{1}{29}\right) = -1$$

$$\boxed{R_F = 29R}$$



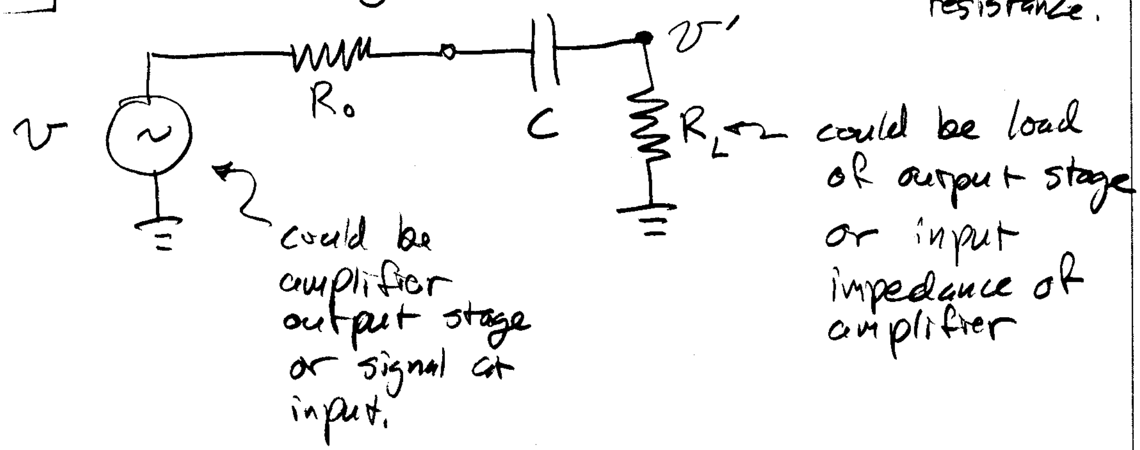
$\boxed{A = R_F/R}$

$\boxed{B = \frac{v_1}{v_0}}$

*German physicist, 1881–1956.

Simplified treatment of coupling capacitors at input or output stages of amplifiers. [Both source and load impedances are important... and must be known for an exact answer. If one is much larger than the other, simplifications are possible.]

$\boxed{\text{High pass}}$ network usually looks like this: R_o = source resistance.



$v = v_o e^{j\omega t}$. At high frequency, this is a resistive divider: $v' \approx \frac{R_L}{R_o + R_L} v$.

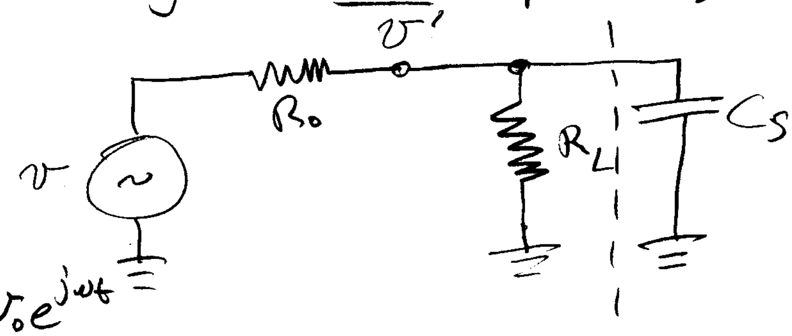
In general, $v' = v \, R_L / (R_o + R_L + 1/j\omega C)$

$$= \left[\frac{v R_L}{R_o + R_L}\right] \frac{(R_o + R_L)}{(R_o + R_L) + 1/j\omega C}.$$

Thus this has a -3dB point relative to the value at high frequency where $|1/j\omega C| = R_o + R_L$

or $f_c = \frac{1}{2\pi (R_o + R_L) C}$.

Similarly, for <u>shunt</u> capacitance, we have:



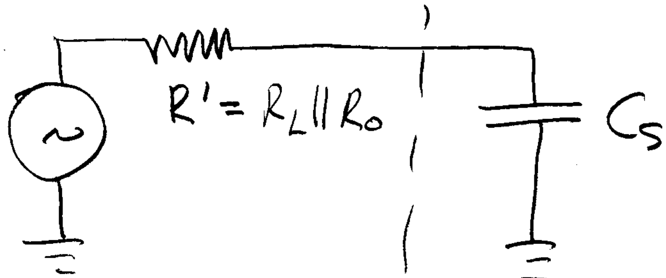
$v = v_0 e^{j\omega t}$

{ Low pass
HF shunted
to ground

At low frequency, $v' = \dfrac{R_L}{R_o + R_L} v$.

The Thevenin equiv. of things to the left of the dotted line is

$v_1 = \dfrac{R_L v}{R_o + R_L}$



so $$f_c = \frac{1}{2\pi R' C_S},$$

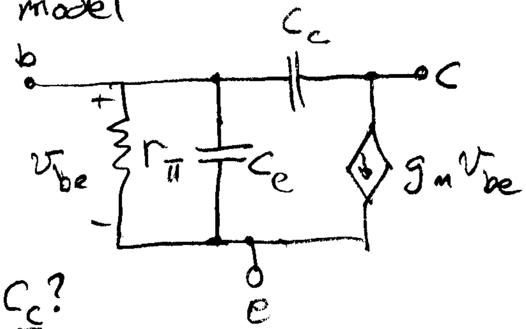
$$R' = \frac{R_L R_o}{R_L + R_o}.$$

If $R_L >> R_o$ (but $R_o \neq 0$), $R' \simeq R_o$.

<u>If</u> we want f_c to be large*, R_o must be small. [We could make R_L small but if R_o is still large, $R_L/(R_o + R_L)$ is small - signal reduced by voltage divider at input].

* e.g. HF response of amplifier.
Note we want f_c to be <u>small</u> for emitter (or source) bypass capacitor.

HF BJT model



$C_c \simeq 5\,pF$
$C_e \simeq 100\,pF$

$g_m = \left.\frac{dI_c}{dV_{BE}}\right|_Q \approx \frac{1}{r_e}$

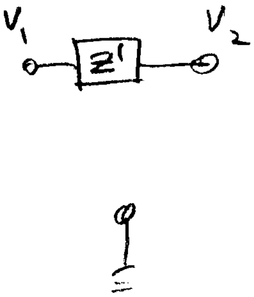
$r_\pi \approx h_{fe} r_e$

What about C_c?

Miller's Theorem:

If $V_2/V_1 = K$

(as for input and output nodes of a voltage amplifier)



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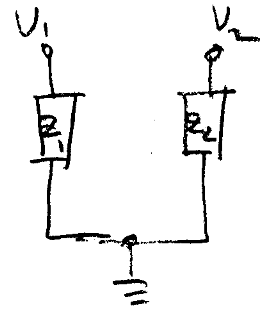


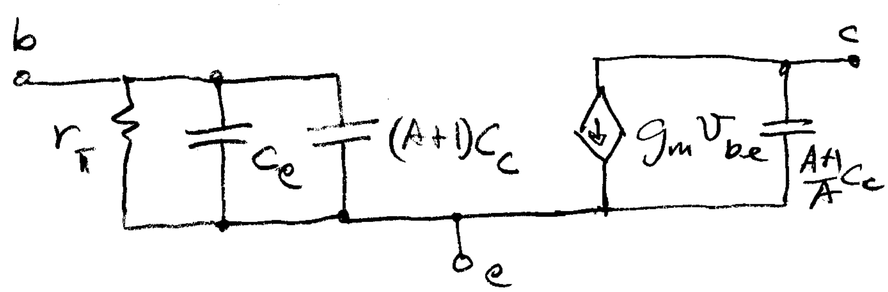
$$Z_1 = \frac{Z'}{1-K}$$

$$Z_1 = \frac{Z'}{1-K}\,,\quad Z_2 = \frac{KZ'}{K-1}$$

If $Z' = 1/j\omega C'$ (capacitor) and $K = -A$ (inverting amplifier)

then $C_1 = (A+1)C'$, $C_2 = \frac{A+1}{A}C'$.

HF BJT model with Miller's thm:

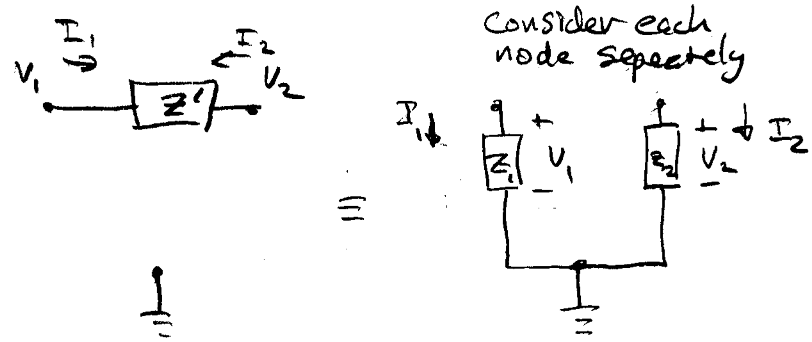


This allows us to seperate input and output networks.

Miller's Theorem proof:

$V_2/V_1 = k$

Consider each node seperately



Same currents, same voltages.

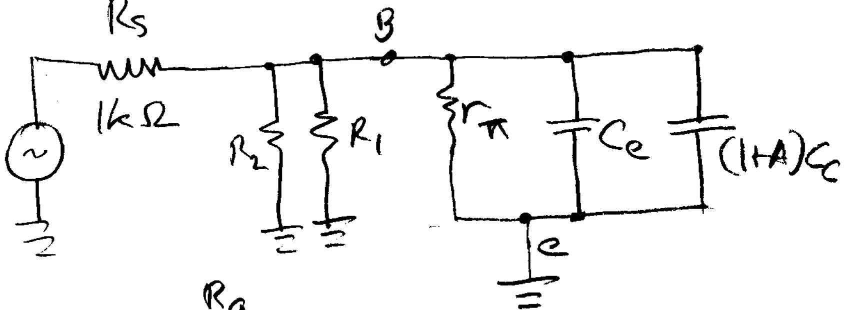
$$I_1 = \frac{V_1 - V_2}{Z'} = \frac{(1-k)V_1}{Z'}$$

$$\frac{V_1}{I_1} \equiv Z_1 = \frac{Z'}{1-k}$$

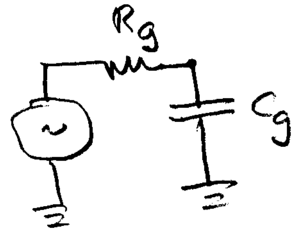
$$I_2 = \frac{V_2 - V_1}{Z'} = \frac{V_2\left(1-\frac{1}{k}\right)}{Z'}$$

$$\frac{V_2}{I_2} \equiv Z_2 = \frac{Z'}{1-\frac{1}{k}} = \frac{kZ'}{k-1}$$

Input stage after applying Miller's thm (high freq. case)



Equiv.:

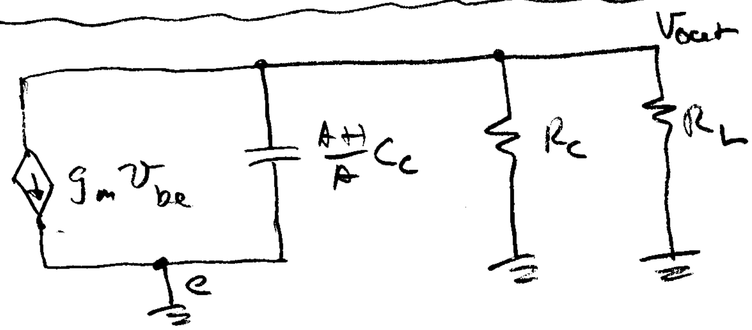


$R_g = R_S \| R_1 \| R_2 \| r_\pi$ $f_c = \dfrac{1}{2\pi R_g C_g}$

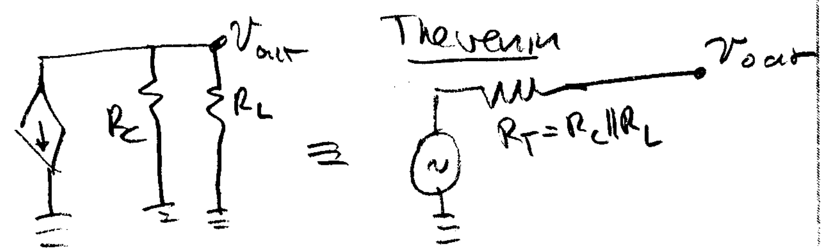
$C_g = C_e + (1+A) C_c$

(Following text notation - g must stand for "lag" (low pass) network)

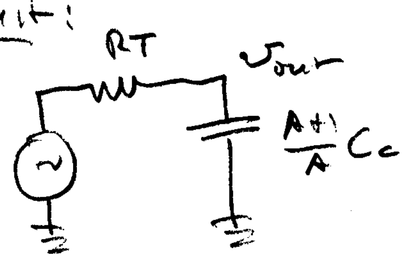
Output stage



Norton:



Circuit:



$$f_c = \frac{1}{2\pi R_T \left(\frac{A+1}{A}\right) C_c}$$

Note that the output circuit displays approx const. gain-bandwidth product (assuming LF extends to ≈ 0)

$BW = f_c$

$A = \frac{R_T}{r_e} = g_m R_T >> 1 \quad (A_v = -A)$

$$\text{gain-BW product} = A \times BW = \frac{R_T}{r_e} \times \frac{1}{2\pi R_T\left(\frac{A+1}{A}\right) C_c} \simeq \frac{1}{2\pi r_e C_c}$$

If the input circuit BW does not limit (i.e., >> this BW) amp. BW, then we get this constant gain-BW product

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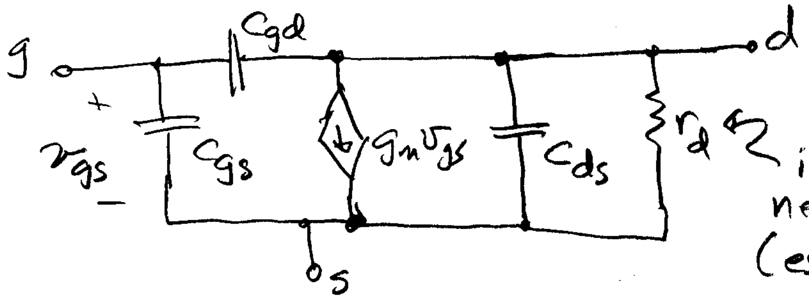
Exact same analysis applies to FETs, except the FET model is slightly different:



if necessary (esp. MOSFET) May neglect for JFET

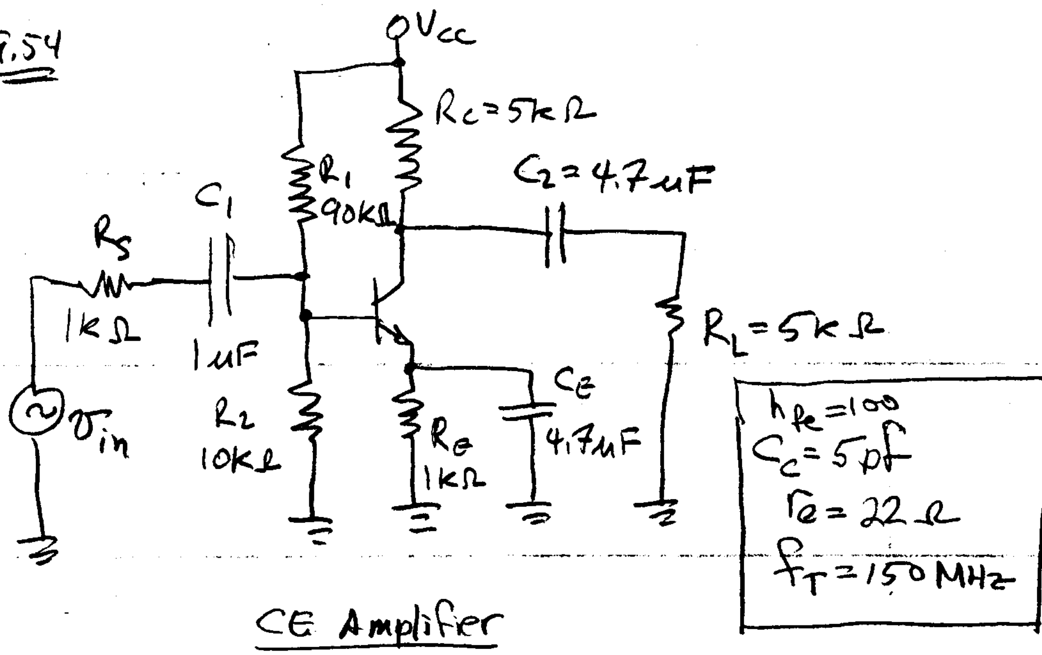
$C_{gs}, C_{gd} \sim 1\text{-}3\,pf$

$C_{ds} \lesssim 1\,pf$

Again, split C_{gd} into (via Miller)

$(A+1)C_{gd}$ in input circuit parallel to C_{gs}, $\left(\frac{A+1}{A}\right)C_{gd}$ in output circuit ∥ to C_{ds}.

9.54



CE Amplifier

Terminology : $C_c \equiv C_{BC}$, $C_e = C_{BE}$

$f_T \equiv$ current gain bandwidth product

$$\simeq \frac{1}{2\pi(C_c + C_e) r_e}$$

(a) Lower cutoff frequency: treat each part separately to see if any one part is dominant:

i) Input circuit (assume $C_E \to \infty$)

$$f_c = \frac{1}{2\pi(R_S + R_1 \| R_2 \| r_\pi) C_1}, \quad r_\pi \simeq h_{fe} r_e = 2200\Omega$$

$$= \frac{1}{2\pi(1k\Omega + 1.77k\Omega) \times 1\mu F} = \underline{58\ Hz}$$

ii) Output circuit :

$$f_c = \frac{1}{2\pi(R_C + R_L) C_2} = \frac{1}{2\pi \times 10k\Omega \times 4.7\mu F} = \underline{3.4\ Hz}$$

iii) Emitter circuit HF f_c due to C_E shunt capacitance, assuming $C_1 \to \infty$ (ineffective below f_c):

R_{equiv} = resistance looking into emitter $= r_e + \frac{R_1 \| R_2 \| R_S}{h_{fe} + 1}$

$$= 22\Omega + \frac{1}{101}(900\Omega) \simeq 31\Omega .$$

$$f_c = \frac{1}{2\pi(R_{equiv} \| R_E) C_E} = \boxed{1.13\ KHz}$$ ← Determines LF −3dB point.

9.54 (b) Upper cutoff frequency

i. Consider input (base) circuit with Miller effect w.r.t. $C_c \equiv C_{CB}$. We need to know mid-band gain, $A \equiv -A_v(\text{midband}) = \frac{R_C \| R_L}{r_e}$ for this circuit

$$A = \frac{2.5\,k\Omega}{22\,k\Omega} = 114$$

The text defines quantities which are equivalent to what was done in class: $\omega_g \equiv 2\pi f_B = \frac{1}{R_g C_g}$

Where $R_g = R_S \| R_p \| r_\pi$ and $R_p = R_1 \| R_2$ (assumes R_E bypassed)

and $C_g = C_e + (1+A)C_c \quad (\equiv C_{BE} + \underbrace{(1-A_v)C_{BC}}_{\text{Miller effect}})$.

Now we need C_e but we have

$$C + C_e = \frac{1}{2\pi f_T r_e} = \frac{1}{2\pi \times 150\text{ MHz} \times 22\,\Omega} = 48\text{ pF}$$

$$C_e = 48\text{pF} - 5\text{pF} = \underline{43\text{ pF}}.$$

$$C_g = 43\text{ pF} + 115 \times 5\text{pF} = \underline{618\text{ pF}}$$

$$f_c = \frac{1}{2\pi R_g C_g} = \frac{1}{2\pi \times 639\,\Omega \times 618\text{pF}} = 403\text{ KHz}.$$

ii. Consider output (collector) circuit with Miller effect.

Again, the text defines quantities which are equivalent to what was done in class.

$$\omega_T \equiv 2\pi f_B = \frac{1}{R_T C_T} \quad \text{where}$$

$$R_T = R_C \| R_L \text{ and } C_T = \underbrace{\frac{1+A}{A}}_{\text{Miller effect}} C_c \left(= \frac{A_v - 1}{A_v} C_{BC}\right)$$

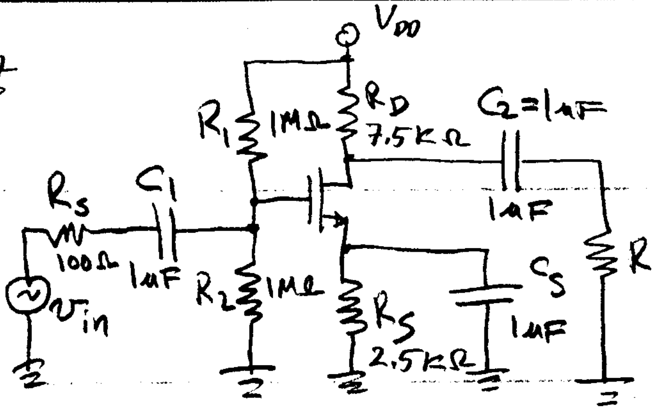
$$f_c = \frac{1}{2\pi \times 2.5\,k\Omega \times \frac{115}{114} \times 5\text{ pF}} = 12.6\text{ MHz}$$

Thus the input circuit dominates and the

<u>HF f_c = 403 kHz.</u>

(c) Bandwidth = 403 KHz − 1 KHz = <u>402 KHz</u>.

9.57



$g_m = 0.8\,m\mho$
$C_{gs} = 4pF$
$C_{gd} = 2pF$
$C_{ds} = 1pF$
$r_d = 10k\Omega$ (see Fig. 9.37)

(a) Lower cutoff frequency:

i) Input circuit (assume $C_S \to \infty$)

$$f_c = \frac{1}{2\pi(R_s + R_1 \| R_2)C_1} = \frac{1}{2\pi(100\Omega + 500k\Omega)\times 1\mu F}$$

$$= \underline{0.3\,Hz}$$

ii) output circuit: assume $C_S \to \infty$ and take r_d into account (see p. 614):

$$f_c = \frac{1}{2\pi(R_D \| r_d + R_L)C_2} = \frac{1}{2\pi \times 9.29k\Omega \times 1\mu F} = \underline{17\,Hz}$$

iii) Source circuit HF f_c due to C_S shunt, taking r_d into account (see pp. 615–616):

$$f_c = \frac{1}{2\pi R_{os} C_S}$$ where $R_{os} = R_S \| \frac{1}{g_m} \| r_d$, according to text, with r_d.

$$= \frac{1}{2\pi \times 769\Omega \times 1\mu F}$$

$$\boxed{f_c = 207\,Hz.}$$ ← determines LF −3dB point.

(b) Upper cutoff frequency: See pp. 621–623. The FET circuit is related to the equiv. HF circuit of the BJT common emitter amplifier (see prob. 9.54 solution):

i) Input $f_c = \frac{1}{2\pi R_g C_g}$,

$R_g = R_s \| R_1 \| R_2 \approx 100\Omega$

$C_g = C_{gs} + (1+A)C_{gd}$

$A = g_m[r_d \| R_D \| R_L]$

$= 0.8\,m\mho \times 2.31k\Omega$

$= 1.85$

$C_g = 4pF + 2.85 \times 2pF = 9.7\,pF$

$$f_c = \frac{1}{2\pi \times 100\Omega \times 9.7pF}$$

$$= \underline{164\,MHz}$$

9.57 (b) ii) Output circuit, again related to CE BJT analysis,

$$f_C = \frac{1}{2\pi R_T C_T} \quad \text{where } R_T = r_d \| R_D \| R_L = 2.31\,k\Omega$$

$$C_T = C_{ds} + \frac{1+A}{A} C_{gd}$$

$$= 1pF + \frac{2.85}{1.85} \times 2pF$$

$$= 4.1\,pF$$

$$= \frac{1}{2\pi \times 2.31k\Omega \times 4.1\,pF}$$

$$\boxed{f_C = 16.8\ MHz} \leftarrow \text{determines HF cutoff}$$

(c) $BW = f_{C_{HIGH}} - f_{C_{LOW}} = 16.8\,MHz$