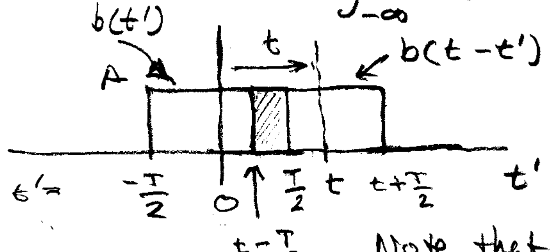
1.

(a) Use the figure below to prove that the <u>convolution</u> of two boxcar functions is a triangle function. [<u>Hint</u>: start by finding the expression for $f(t)$, $t>0$.]

Boxcar: $b(t) = \begin{cases} A & -T/2 < t < T/2 \\ 0 & \text{otherwise} \end{cases}$

<u>Evaluate</u>: $f(t) = b * b \equiv \int_{-\infty}^{\infty} b(t')\, b(t-t')\, dt'$




Note that $b(t')\,b(t-t') = 0$ everywhere except for the shaded region. Only this region contributes to the integral.

(b) Use the convolution theorem to find an expression for the Fourier transform of the triangle function from part (a).* <u>Hint</u>: the F.T. of the boxcar, $b(t)$, is

$$B(f) = AT\,\frac{\sin \pi T f}{\pi T f}.$$

(c) Find the one-sided spectral density for the triangle function.

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* you don't need the answer of (a) to do (b) and (c).

(a) Refer to the figure.

$$f(t) = \int_{-\infty}^{\infty} b(t')\, b(t-t')\,dt' \quad , \text{ consider } t \geq 0$$

$$= \int_{t-T/2}^{T/2} A^2\,dt' = A^2\left[\frac{T}{2} - \left(t - \frac{T}{2}\right)\right]$$

$$= A^2\left[T - t\right], \quad 0 \leq t \leq T.$$

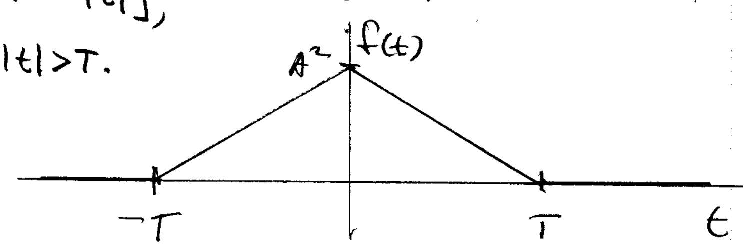
For $t \leq 0$,

$$f(t) = \int_{-\frac{T}{2}}^{t+\frac{T}{2}} A^2\,dt' = A^2\left[t + \frac{T}{2} - \left[-\frac{T}{2}\right]\right]$$

$$= A^2\left[T + t\right], \quad -T \leq t \leq 0.$$

We can combine these expressions as follows:

$$f(t) = \begin{cases} A^2\left[T - |t|\right], & -T \leq t \leq T \\ 0, & |t| > T. \end{cases}$$




(b) Since $f(t) = b * b$,

the F.T., $F(f) = B \cdot B$ where $B = $ F.T. of $b(t)$,

$$= A^2 \cancel{T^2} \frac{\sin^2 \pi T f}{\pi^2 \cancel{T^2} f^2} = A^2 \frac{\sin^2 \pi T f}{\pi^2 f^2}.$$

(c) $P(f) = 2\left|F(f)\right|^2 = 2A^4 \dfrac{\sin^4 \pi T f}{\pi^4 f^4}, \quad 0 \leq f < \infty.$