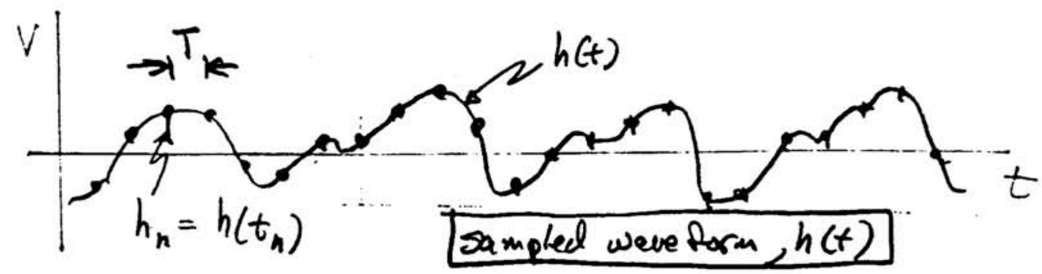
2) Sampled signals, sampling theorem and aliasing:



$$h(t) \rightarrow h_n \quad \begin{cases} n = 0, 1, \ldots, N-1 \quad (N \text{ samples}) \\ \text{at time } t = nT \end{cases}$$

(measure time in units of T)

3 Define Nyquist critical frequency $f_c = \frac{1}{2T}$
(half the sampling frequency)

Note that sine wave at f_c has two samples/cycle (minimum necessary to recognize the component — and if phase were just right (or just wrong, rather) could not see it at all.

4 Sampling Theorem

If $h(t)$ (continuous fcn) is sampled at intervals T and is band-width limited to frequencies less than or equal to f_c, i.e., $H(f) = 0$ for all $|f| > f_c$, then $h(t)$ can be reconstructed uniquely from the samples. (But note in principle that the number of samples may be very large!). In particular,

$$h(t) = T \sum_{n=-\infty}^{\infty} h_n \frac{\sin[2\pi f_c(t - nT)]}{\pi(t - nT)} .$$

$$\left[\text{shorthand: } \frac{\sin x}{x} \equiv \text{sinc } x\right]$$

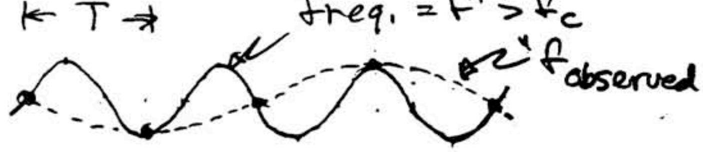
* Numerical Recipes (see Refs)

5 **Aliasing:**

If $h(t)$ is not band-width limited appropriately, all the frequency components outside

$$-f_c < f < f_c$$

are mapped into this range (aliasing):



$$\left\{ \begin{array}{l} f' = \dfrac{3}{4T} \\ \text{samples look like} \\ f_{obs} = \dfrac{1}{4T} \end{array} \right.$$

In general, $\left\{ \begin{array}{ll} f_{observed} = f' - nf_c & (\text{even } n) \\ \quad\quad\quad\;\; = -f' + (n+1) f_c & (\text{odd } n) \end{array} \right\}$ some unique n will bring f_{obs} into $0 < f_{obs} < f_c$

Check:

Here, $n = 1$ and $f_{obs.} = -f' + 2f_c = -\frac{3}{4T} + \frac{1}{T} = \frac{1}{4T}$ (ok)

6 In general, may need to filter frequencies — e.g., for audio response to 17 kHz with sample frequency of 44 kHz, must apply a sharp cutoff filter to eliminate $f > f_c = 22$ kHz... although little power remains in this part of the spectrum.

References: Press, et al., Numerical Recipes, Ch. 12 (N.R.)

Brigham, The Fast Fourier Transform and its Applications