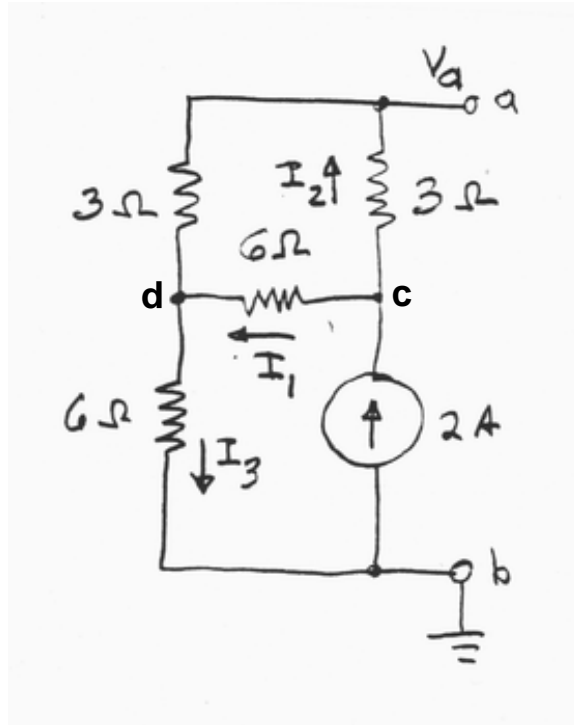


# Physics 116A: Solution to Exam I from 2/3/2000

Draft 10/22/2000; send D. Pellett e-mail if you find bugs.



1. (a) At node c,  $I_1 + I_2 = 2 \text{ A}$ ; at node d,  $I_1 + I_2 = I_3$ . Thus,  $I_3 = 2 \text{ A}$ .  
The network between nodes c and d is a current divider and each branch has a total resistance of  $6 \Omega$ . Thus each branch carries half the total current and  $I_1 = I_2 = 1 \text{ A}$ .
- (b)  $V_{oc} = V_a = I_2 \times 3 \Omega + I_3 \times 6 \Omega = 3 \text{ V} + 12 \text{ V} = 15 \text{ V}$ .
- (c) The Thévenin equivalent of the circuit connected to the nodes a and b consists of the voltage source just found,  $V_{oc} = 15 \text{ V}$ , in series with the resistance,  $R_0$ , determined from the network between a and b resulting from setting the independent sources to zero.
  - i. Setting the current source to zero results in (b) an open circuit in its place.  
Thus  $R_0 = 3 \Omega \parallel 9 \Omega + 6 \Omega = 2.25 \Omega + 6 \Omega = 8.25 \Omega$ .
- (d) Now use the Thévenin equivalent of the circuit. The output voltage,  $V_a$ , with a  $6.75 \Omega$  resistor,  $R_L$ , connected between a and b can be found using the voltage divider formula since the circuit consists of  $V_{oc}$ ,  $R_0$  and  $R_L$  in series.

$$V_a = 15 \text{ V} (6.75 \Omega / (8.25 \Omega + 6.75 \Omega)) = 6.75 \text{ V}.$$

2. (a)  $V_{in} = i_{in}R_B + (i_{in} + \beta i_{in})R_E = i_{in}(R_B + (\beta + 1)R_E)$ .  
 (b)  $R_{eff} \equiv V_{in}/i_{in} = R_B + (\beta + 1)R_E$ .  
 (c)  $R_{eff} = 2500 \Omega + 101 \times 200 \Omega = 22.7 \text{ K}\Omega$ .
3. (a) There is a virtual ground at the inverting input of the op-amp since the non-inverting input is grounded and there is negative feedback.  
 (b) Let the (complex) current to the right through  $R$  be  $\mathbf{I}$  (and assume the angular frequency is  $\omega$ ).  $\mathbf{V}_{in} = \mathbf{I}R$ . Also, since no current flows into the op-amp inverting input,  $\mathbf{V}_{out} = -\mathbf{I}\mathbf{Z}$  where

$$\mathbf{Z} = \mathbf{Z}_C || R = \frac{R/\mathbf{j}\omega C}{R + 1/\mathbf{j}\omega C} = \frac{R}{1 + \mathbf{j}\omega RC}.$$

$$\mathbf{H}(\mathbf{j}\omega) = \mathbf{V}_{out}/\mathbf{V}_{in} = -\mathbf{Z}/R = -1/(1 + \mathbf{j}\omega RC) = -1/(1 + \mathbf{j}\omega/\omega_0),$$

with  $\omega_0 \equiv 1/RC$ .

$$(c) |\mathbf{H}(\mathbf{j}\omega)| = \sqrt{\mathbf{H}^*\mathbf{H}} = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}.$$

- (d) i. For  $\omega \gg \omega_0$ , the quadratic term in the denominator is much larger than 1 and  $|\mathbf{H}(\mathbf{j}\omega)| \approx \omega_0/\omega$ .  
 ii. For  $\omega \ll \omega_0$ , the quadratic term in the denominator is much less than 1 and  $|\mathbf{H}(\mathbf{j}\omega)| \approx 1$ .  
 This is a low-pass filter.

$$(e) V_{in} = 2 \text{ V} \cos \omega_0 t,$$

$$\mathbf{H}(\mathbf{j}\omega) = -1/(1 + \mathbf{j}) = \frac{1 \angle 180^\circ}{\sqrt{2} \angle 45^\circ} = 0.707 \angle 135^\circ,$$

$$V_{out} = 0.707 \times 2 \text{ V} \cos(\omega_0 t + 135^\circ) = 1.4 \text{ V} \cos(\omega_0 t + 135^\circ).$$

The amplitude of the output voltage is 1.4 V and the output leads the input by  $135^\circ$  at this frequency.

4. (a) We can find  $\mathbf{V}_{out}$  and  $\mathbf{H}(\mathbf{j}\omega)$  using the voltage divider formula.

$$\mathbf{V}_{out} = \frac{\mathbf{Z}_L \mathbf{V}_{in}}{R + \mathbf{Z}_C + \mathbf{Z}_L},$$

$$\mathbf{H}(\mathbf{j}\omega) = \mathbf{V}_{out}/\mathbf{V}_{in} = \frac{\mathbf{j}\omega L}{R + 1/(\mathbf{j}\omega C) + \mathbf{j}\omega L} = \frac{-\omega^2 LC}{1 + \mathbf{j}\omega RC - \omega^2 LC}.$$

$$(b) \text{ With } \omega_R \equiv 1/\sqrt{LC},$$

$$\mathbf{H}(\mathbf{j}\omega) = \frac{-(\omega/\omega_R)^2}{1 - (\omega/\omega_R)^2 + \mathbf{j}(\omega/\omega_R)R\sqrt{C/L}}.$$

- (c) Use of the complex frequency,  $\mathbf{s}$ , extends the domain of the transfer function,  $\mathbf{H}$ , from the imaginary axis to the entire complex plane:  $\mathbf{H}(\mathbf{j}\omega) \rightarrow \mathbf{H}(\mathbf{s})$ , where  $\mathbf{s} \equiv \sigma + \mathbf{j}\omega$ . This is useful for classifying filter response and for dealing with non-sinusoidal waveforms. We find  $\mathbf{H}(\mathbf{s})$  by following our previous analysis but using  $\mathbf{s}$  instead of  $\mathbf{j}\omega$ . For example,  $\mathbf{Z}_L = \mathbf{s}L$ . See Secs. 5.3 and 5.4 in the text for details. Following this recipe, we get

$$\mathbf{H}(\mathbf{s}) = \mathbf{s}L / (R + \mathbf{s}L + 1/\mathbf{s}C) = \frac{LC\mathbf{s}^2}{1 + RC\mathbf{s} + LC\mathbf{s}^2}.$$

- (d)  $\mathbf{H}(\mathbf{s})$  has zeros where the numerator vanishes (if in fact it does), so this  $\mathbf{H}(\mathbf{s})$  has zeros at  $\mathbf{s} = 0$ .
- (e) In Sec. 5.2 of the text, it was shown that for a sharp resonance, the *bandwidth*, BW (defined as the full width of the resonance peak (plotted vs.  $\omega$ ) at the half-power point), is given by  $\text{BW} = \omega_R/Q$ , where  $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$  for this series LRC circuit. Thus the problem is asking for the bandwidth:

$$\text{BW} = \omega_R/Q = 2\pi f_R/Q = 6.28 \times 11.3 \text{ KHz}/70.7 = 1.00 \times 10^3 \text{ rad/s} = 160\text{Hz}.$$

- (f) The denominator of  $\mathbf{H}(\mathbf{s})$  in this case is a quadratic in  $\mathbf{s}$  which has complex conjugate roots in the left half of the complex plane for the values given in part (e). You can verify this by putting in the values and solving. Simple roots in the denominator correspond to poles of the complex function,  $\mathbf{H}(\mathbf{s})$ , (so called because  $\mathbf{H}(\mathbf{s})$  becomes infinite, although the term relates to only certain types of infinities in complex function theory). These poles are at complex conjugate points in the left half-plane, answer (ii).