

Physics 9HD Midterm 1 Review, Fall 2006

Note: All units in CGS, unless otherwise specified.

e = charge of electron = 4.8×10^{-10} esu = 1.6×10^{-19} coul

Electric Forces and Fields, Gauss's Law

Coulomb's Law : $\vec{F} = \frac{Q_1 Q_2}{r^2} \hat{r}$, Q_i in esu, r in cm, F in dynes

Electric Field (in dyne/esu = esu/cm²): $\vec{E} = \frac{Q}{r_1^2} \hat{r}_1$ for point charge, $\vec{F} = q\vec{E}$

Electric Field between 2 parallel plates : $\vec{E} = 4\pi\sigma$, where σ =surface charge density = Q/A

Superposition of electric fields: $\vec{E}_{Tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$

\vec{E} field of a charge distribution:

$$\vec{E}(x, y, z) = \int \frac{\rho(x', y', z') \hat{r} dx' dy' dz'}{r^2} = \int \frac{\rho(x', y', z') \hat{r} dx' dy' dz'}{(x-x')^2 + (y-y')^2 + (z-z')^2}, \text{ where } \hat{r} \text{ is a unit vector pointing from } (x', y', z') \text{ to } (x, y, z).$$

Gauss's Law: $\oint \vec{E} \cdot d\vec{a} = 4\pi Q_{encl} = 4\pi \int \rho dv = 4\pi \int \sigma da = 4\pi \int \lambda dl$

Differential form: $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$

If you are using Gauss's Law to find electric field, you must choose surface so that \vec{E} is constant on the surface. (Conversely, if the problem is not symmetric enough that you can identify a surface over which you expect \vec{E} to be constant, Gauss's Law will not help you to find the \vec{E} field.) In addition, you must integrate function ρ over the volume if ρ is not constant, i.e., find $\int \rho dv = \int \sigma da = \int \lambda dl$, whichever is appropriate for geometry of the problem.

Know how to derive \vec{E} field for the following geometries: infinite sheet of charge, between two infinite flat parallel plates, inside and outside spherical distribution of charge $\rho(r)$, inside and outside cylindrical distribution of charge $\rho(r)$.

$\vec{E} = 0$ inside body of conductor.

$\vec{E} = 0$ inside a cavity in a conductor, if there is no charge in the cavity.

For conductor with a cavity containing charge +Q, induced charge on surface nearest the cavity will be -Q.

Electric Potential and Electric Energy

Electric potential V or ϕ (in statvolts), $\Delta\phi = -\int \vec{E} \cdot d\vec{l}$, $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$, $\vec{E} = -\vec{\nabla}\phi$

The line integral of $\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}$ is independent of the path between P_1 and P_2 .

$\therefore \oint_C \vec{E} \cdot d\vec{l} = 0$ for any closed path C.

Potential energy (in ergs) of two charges Q_1 and Q_2 , separated by distance r, $U = \frac{Q_1 Q_2}{r}$

Potential energy gained by charge q in going through potential difference $\Delta\phi$ is $U = q\Delta\phi$

Electric potential at distance r from point charge q is $V(r) = \frac{q}{r}$

Electric potentials superpose: $V_{eq} = V_1 + V_2 + V_3 + \dots + V_n$

Potential of a charge distribution:

$$\phi(x, y, z) = \int \frac{\rho(x', y', z') dx' dy' dz'}{r} = \int \frac{\rho(x', y', z') dx' dy' dz'}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$

1 statvolt = 300 V

1 Electron Volt is the energy gained by an electron in going through a potential of 1V.

Since the charge on an electron is 1.6×10^{-19} coul, $1\text{eV} = 1.6 \times 10^{-19}$ j

Energy in electric field: $U = \int \frac{E^2}{8\pi} dv$,

Gradient, divergence, curl, Laplacian

$$\vec{\nabla} \phi = \hat{x} \frac{\partial \phi}{\partial x} + \hat{y} \frac{\partial \phi}{\partial y} + \hat{z} \frac{\partial \phi}{\partial z}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\vec{\nabla} \times \vec{E} = \hat{x} \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \hat{y} \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + \hat{z} \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

Laplacian

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

$$\int_C (\vec{\nabla} \phi) \cdot d\vec{s} = \phi(b) - \phi(a)$$

$$\oint_C \vec{A} \cdot d\vec{s} = \int_S (\text{curl} \vec{A}) \cdot d\vec{a} \quad \text{Stokes' Theorem}$$

$$\oint_S \vec{F} \cdot d\vec{a} = \int_V (\text{div} \vec{F}) dv \quad \text{Divergence Theorem}$$

Poisson's Equation $\nabla^2 \phi = -4\pi\rho$

Laplace's Equation $\nabla^2 \phi = 0$

Uniqueness Theorem for solution of Laplace's equation with potentials given on conductors:

There is only one solution. If you find a possible solution, it is the **only** solution.

Method of Images-- for solving Laplace's equation—used to solve the problem of one or more point charges in the presence of boundary surfaces, e.g., conductors which are grounded or a fixed potential.

Put image charges outside the region of interest to simulate the boundary conditions.

Replace the actual problem with boundaries by enlarged region with image charges and no boundaries. Note $\nabla^2 \phi = 0$ from the image charges in the volume of interest.

The force between one point charge and the conductor is due to the force between the charge and the induced charge on the conductor. This force is equal to the force between the point charge and the image charge.

Capacitance

$Q=CV$, or $C = \frac{Q}{\Delta\phi}$ (in cm) Capacitance depends only on geometry (and presence of dielectric material)

Calculate capacitance by using Gauss's Law to compute \vec{E} field with charge Q on one conductor, -Q on the other conductor, then compute potential difference (voltage) by doing line integral over \vec{E} field, i.e., $\Delta\phi = -\int \vec{E} \cdot d\vec{l}$, then use $C=Q/\Delta\phi$ to find capacitance.

Standard geometries: Parallel plates $C = \frac{A}{4\pi s}$

Know how to calculate capacitance of concentric spheres and coaxial cable.

Energy in capacitor, $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

Capacitors in series: Same magnitude of charge on all electrodes,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

Capacitors in parallel: Same voltage across each capacitor, $C_{eq} = C_1 + C_2 + C_3 + \dots + C_n$