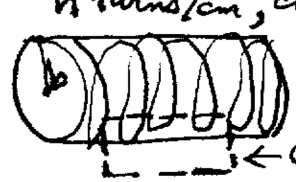
Physics 9HD, Midterm 2, Nov 28, 2007 Solutions

1. n turns/cm, current I



Ampere's Law $\int \vec{B}\cdot d\vec{s} = \frac{4\pi I_{encl}}{c}$

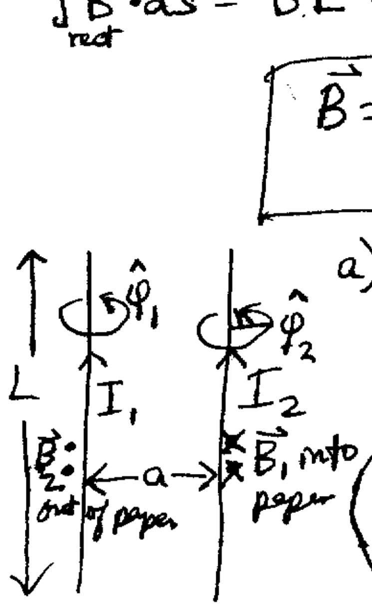
← Consider rectangle of length L

No field outside solenoid, no field on parts of rectangle ⊥ cylinder side.

$$\int_{rect} \vec{B}\cdot d\vec{s} = BL = 4\pi \frac{NI}{c}$$, where N is number of turns in length L, i.e. $N = nL$

$$\boxed{\vec{B} = 4\pi \frac{NI}{Lc} = \frac{4\pi n I}{c}\hat{z}}$$, where $\hat{z}$ is along axis of solenoid

2. a) $\vec{B}$ field from one wire at position of the other is given by $\vec{B} = \frac{2I}{ac}\hat{\varphi}$ where $\hat{\varphi}$ is given by right hand rule curling around wire when thumb along I.



(Derivation of field by Ampere's law not required:

$$\int \vec{B}\cdot d\vec{s} = 4\pi \frac{I_{encl}}{c}$$

$$B(2\pi r) = \frac{4\pi I}{c} \Rightarrow B = \frac{2I}{rc}$$)

$$\boxed{F_{on 2} = \frac{B_1 I_2 L}{c} = \frac{2I_1 I_2 L}{ac^2} \qquad F_{on 1} = \frac{B_2 I_1 L}{c} = \frac{2I_2 I_1 L}{ac^2}}$$

Force is attractive, as proved below.

Forces are equal and opposite, as expected from Newton's 3rd law.

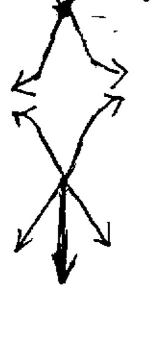
b) By right hand rule in diagram above, B_1 is into paper at position of wire 2. Then $\vec{F} = q\frac{\vec{v}}{c}\times\vec{B}$, where $\vec{v}$ is along direction of I_2, gives $F_{on 2}$ to left.

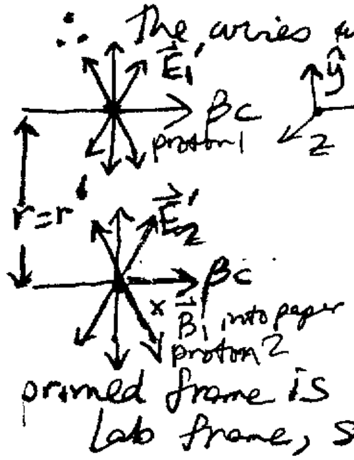
Similarly, right hand rule gives B_2 out of paper at position of wire 1. Then cross product $\vec{v}\times\vec{B}$ gives $F_{on 1}$ to right.

∴ The wires attract, as expected.

3.

Another picture of Interacting Field lines





a) $$E' = \frac{Q}{r'^2}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$$

θ' is angle between charge motion and direction of observation, so $\theta' = \frac{\pi}{2}$, $\sin\theta' = 1$.

$$\boxed{E'} = \frac{Q}{r^2}\frac{1-\beta^2}{(1-\beta^2)^{3/2}} = \boxed{\frac{\gamma Q}{r^2}}$$ direction away from particle causing field

primed frame is lab frame, shown here

3b) $\vec{F}_{on\,1} = \frac{Q^2}{r^2}\hat{y}$ in proton rest frame (repulsive force)

$$\boxed{\vec{F}_{\perp\,on\,1}'} = \frac{F_\perp}{\gamma} = \boxed{\frac{Q^2}{\gamma r^2}\hat{y}} \quad , \quad \vec{F}_{\perp\,on\,2}' = -\frac{Q^2}{\gamma r^2}\hat{y} \quad \text{(equal and opposite)}$$

c) Consider field of E_1 at position of proton 2

$$\vec{E}_1' = -\frac{\gamma Q}{r^2}\hat{y} \qquad B_z' = \gamma(B_z + \beta E_y)$$

Reverse sign because frame F' moves in -x direction with speed v as viewed from F.

$$, \quad \underbrace{\boxed{B_z'}}_{\text{from 1 at 2}} = -\gamma\beta E_y = \boxed{-\gamma\beta\frac{Q}{r^2}}\,,$$

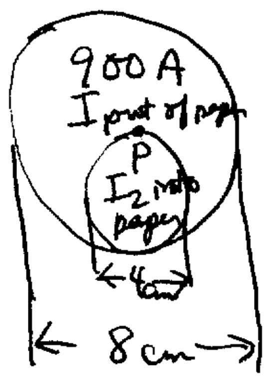
i.e. $\vec{B}_1$ field into paper at position of proton 2.

$$|\vec{F}_{mag}| = \left|Q\frac{\vec{v}}{c}\times\vec{B}\right| = Q\beta B = \frac{Q^2\gamma\beta^2}{r^2} \quad \text{attractive force}$$

$$F_{Tot} = \overset{\text{repulsive}}{F_{electric}} + \overset{\text{attractive}}{F_{mag}} = \frac{\gamma Q^2}{r^2} - \frac{Q^2\gamma\beta^2}{r^2} = \gamma(1-\beta^2)\frac{Q^2}{r^2}$$

$$\boxed{F_{Tot} = \frac{Q}{\gamma r^2}} \quad \text{repulsive force}$$

4.



Use superposition.

Consider current going into paper in small cylinder.

$$I_1 = \int \vec{J}\cdot d\vec{a}$$

$$\Rightarrow J = \frac{I_1}{\pi(4^2-2^2)} = \frac{I_1}{12\pi}$$

current in small cylinder

$$\rightarrow I_2 = J(\pi)(2^2) = 4\pi J = \frac{4\pi I}{12\pi} = \frac{1}{3}I_1$$

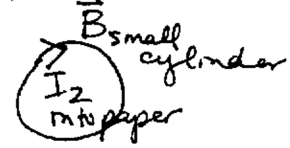
$$= 300\text{ A}.$$

Field from big cylinder of current come out of paper through all of it (including hole) $= 0$ because P is on axis.

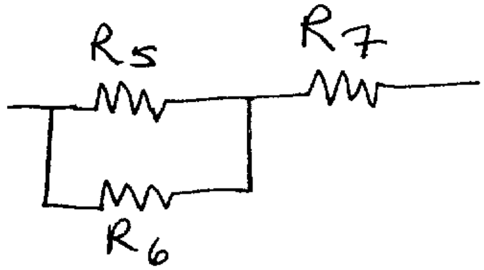
$$\text{Field of small cylinder at P} = \frac{2I_2}{rc} = \frac{2(300\text{ A})\left(\frac{4.8\times10^{-10}\text{ esu}}{1.6\times10^{-19}\text{ coul}}\right)}{2\text{ cm}\left(3\times10^{10}\frac{\text{cm}}{\text{sec}}\right)}$$

$$\vec{B}_{at\,P} = \boxed{30\text{ gauss}}$$

The field points to the right.



5. a)



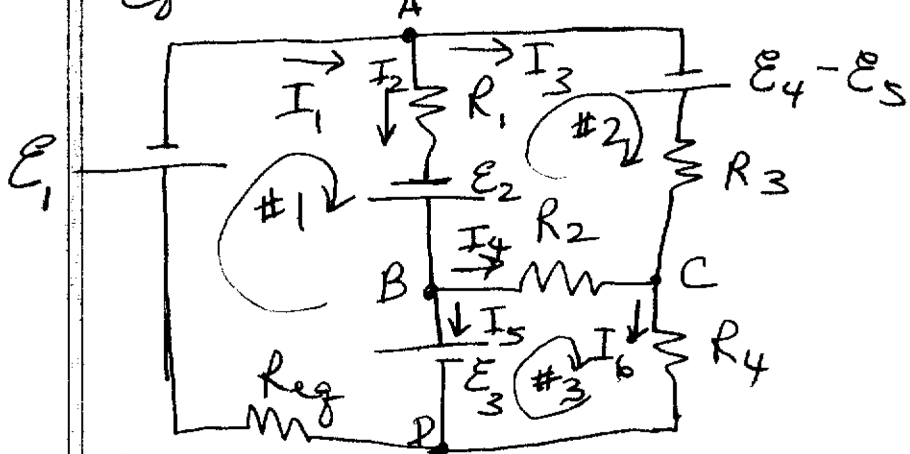
Let R_{11} be parallel resistance of R_5 + R_6.

$$\frac{1}{R_{11}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{R_6 + R_5}{R_5 R_6}$$

$$R_{11} = \frac{R_5 R_6}{R_5 + R_6}$$

$$\boxed{R_{eq}} = R_{11} + R_7 = \boxed{\frac{R_5 R_6}{R_5 + R_6} + R_7}$$

Equivalent circuit is:



Node A : $I_1 = I_2 + I_3$

Node B : $I_2 = I_4 + I_5$

Node C : $I_3 + I_4 = I_6$

Loop #1 : $-\mathcal{E}_1 - I_2 R_1 + \mathcal{E}_2 - \mathcal{E}_3 - I_1 R_{eq} = 0$

Loop #2 : $-\mathcal{E}_2 + I_2 R_1 + (\mathcal{E}_4 - \mathcal{E}_5) - I_3 R_3 + I_4 R_2 = 0$

Loop #3 : $\mathcal{E}_3 - I_4 R_2 - I_6 R_4 = 0$

Other loop equations are possible, but the six equations above are an independent set.

Node D : $I_5 + I_6 = I_1$ is not independent of the 3 node eq. above.