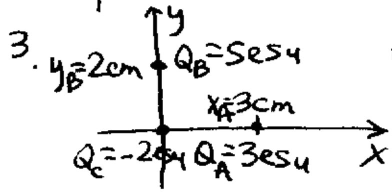
# Physics 9HD - Practice Problems for Midterm I - Solutions

1. positive
2. parabola
3. 


At origin, $\vec{E}_A = \frac{Q_A}{(x_A)^2}(-\hat{x})$, $\vec{E}_B = \frac{Q_B}{(y_B)^2}(-\hat{y})$

$$\vec{E}_{Tot} = -\frac{Q_A}{(x_A)^2}\hat{x} - \frac{Q_B}{(y_B)^2}\hat{y} = \left(-\frac{3}{9}\hat{x} - \frac{5}{4}\hat{y}\right)\frac{esu}{cm^2} = \boxed{\left(-\frac{\hat{x}}{3} - \frac{5}{4}\hat{y}\right)\frac{esu}{cm^2}}$$

$$|\vec{E}_{Tot}| = \sqrt{\frac{1}{9} + \frac{25}{16}} = \boxed{1.29\,\frac{esu}{cm^2}}$$

$$\vec{F} = q\vec{E}_{Tot} = (-2esu)\left(-\frac{\hat{x}}{3} - \frac{5}{4}\hat{y}\right)\frac{esu}{cm^2} = \boxed{\left(\frac{2}{3}\hat{x} + \frac{5}{2}\hat{y}\right)\text{dyne}}$$

$$|\vec{F}| = \sqrt{\frac{4}{9} + \frac{25}{4}}\ \text{dyne} = \boxed{2.59\ \text{dyne}} = q|E_{Tot}| = (2\,esu)\left(1.29\frac{esu}{cm^2}\right) = 2.58\ \text{dyne}$$

4. 


$\rho = Cr^2$ for $r \le a$

For $r<a$, Consider Gaussian cylinder, radius $r<a$, length $L$.

Gauss's law: $\int \vec{E}\cdot d\vec{a} = 4\pi Q_{encl}$

By symmetry, $E$ is radial, $E(2\pi rL) = 4\pi\int_0^r \rho(r')(2\pi r'dr')L$

$$E(2\pi rL) = 4\pi\int_0^r 2\pi C r'^3 dr' L = 8\pi^2 CL\frac{r^4}{4}$$

$r<a$, $\boxed{\vec{E} = \pi C r^3 \hat{r}}$

For $r>a$, consider larger Gaussian cylinder, radius $r>a$, length $L$.

$$\int \vec{E}\cdot d\vec{a} = 4\pi Q_{encl} = E(2\pi rL) = 4\pi\int_0^a (Cr'^2)(2\pi r'dr')L = 8\pi^2 CL\frac{a^4}{4}$$

$r \ge a$, $\boxed{\vec{E} = \pi C\frac{a^4}{r}\hat{r}}$

Now find the potential for $r<a$, $\varphi(r) - \varphi(0) = -\int_0^r \vec{E}\cdot d\vec{s} = -\pi C\int_0^r r'^3 dr' = -\pi C\frac{r^4}{4}$

So $r<a$, $\boxed{\varphi(r) = \pi C\frac{r^4}{4}}$, $\varphi(a) = \pi C\,a^4/4$

For $r>a$, $\varphi(r) - \varphi(a) = -\int_a^r \vec{E}\cdot d\vec{s} = -\int_a^r \pi C\frac{a^4}{r}dr = -\pi C a^4 \ln\frac{r}{a}$

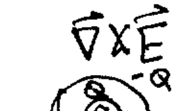
$$\varphi(r) = \varphi(a) - \pi C a^4 \ln\frac{r}{a} = \pi C\frac{a^4}{4} - \pi C a^4\ln\frac{r}{a}$$

$r \ge a$, $\boxed{\varphi(r) = \pi C a^4\left[\frac{1}{4} - \ln\frac{r}{a}\right]}$

5. $\varphi = 3xy + x^2 - y^2$, $\vec{E} = -\vec{\nabla}\varphi = -(3y+2x)\hat{x} - (3x-2y)\hat{y}$

$\nabla^2\varphi = 2-2 = 0$, i.e. satisfies Laplace's equation. $\therefore$ $\boxed{\rho = 0}$

$\vec{\nabla}\times\vec{E} = \hat{x}\left(\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y\right) + \hat{y}\left(\frac{\partial}{\partial z}E_x - \frac{\partial}{\partial x}E_z\right) + \hat{z}\left(\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\right) = \hat{z}(-3+3) = 0$ $\therefore$ possible $\vec{E}$ field

6. 


$\vec{E} = \frac{Q}{r^2}\hat{r}$ between spheres, $\varphi(a) - \varphi(b) = -\int_b^a \frac{Q}{r^2}dr = \left[\frac{Q}{r}\right]_b^a = \frac{Q}{a} - \frac{Q}{b}$

$$C = \frac{Q}{\Delta\varphi} = \frac{Q}{\frac{Q}{a} - \frac{Q}{b}} = \boxed{\frac{ab}{b-a}}$$

7. 



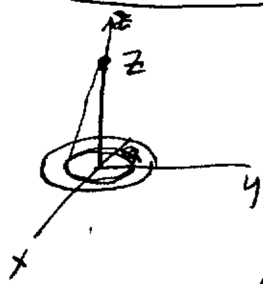
Charge of $-Q$ is induced on inner surface of conductor

$$-Q + Q_{\text{outer surface}} = -5Q$$

$$\therefore\ Q_{\text{outer surface}} = -4Q$$

Outside the conductor, the field looks like that of a point charge $(-4Q)$ at origin.

$$\boxed{\vec{E} = -\frac{4Q}{r^2}\hat{r}}$$

8. 



Use cylindrical coordinates. Consider contribution to the potential at $z$ from a ring of radius $r$, width $dr$ in $xy$ plane.

$dQ_{ring} = \sigma(2\pi r\,dr)$ , $\sigma = \frac{Q}{\pi r^2} = \text{const.} = \text{surf. charge density}$

$$\varphi(z) = \boxed{\int_0^a \frac{\sigma(2\pi r\,dr)}{\sqrt{r^2+z^2}}} = 2\pi\sigma\left[\sqrt{r^2+z^2}\right]_0^a = \boxed{2\pi\sigma\left(\sqrt{a^2+z^2} - z\right)}$$

integral form

$$\vec{E} = -\vec{\nabla}\varphi = -\hat{z}\frac{\partial}{\partial z}\varphi = -\hat{z}(2\pi\sigma)\int_0^a \frac{r\,dr\,(-\frac{1}{2})(2z)}{\left[r^2+z^2\right]^{3/2}} = \boxed{\hat{z}(2\pi\sigma)z\int_0^a \frac{r\,dr}{(r^2+z^2)^{3/2}}}$$

integral form

We actually have a closed form for $\varphi(z)$, so we can obtain

$$\vec{E} = -\vec{\nabla}\varphi = -\hat{z}\frac{\partial}{\partial z}\varphi = \boxed{-\hat{z}(2\pi\sigma)\left[\frac{z}{\sqrt{a^2+z^2}} - 1\right]}$$