Physics 9HD, Midterm 1, October 29, 2007

You are permitted to use one 8 /1/2" x 11" sheet of paper with formulas. You will not need your calculator. <u>MAXIMUM SCORE IS 100 POINTS</u>. SHOW ALL WORK FOR PARTIAL CREDIT.

Useful formulas: In cylindrical coordinates, $\vec{\nabla}\psi(r,\phi,z) = \hat{r}\frac{\partial\psi}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial\psi}{\partial\phi} + \hat{z}\frac{\partial\psi}{\partial z}$

and $\nabla^2\psi(r,\phi,z) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2} + \frac{\partial^2\psi}{\partial z^2}$

1. (10 pts) (a) Consider a neutrally charged, perfect conductor which is a perfect cube, with a spherical cavity in the middle. Somehow, a point charge of +Q is placed into the center of the cavity. What is the induced charge on the inside of the spherical cavity? What is the induced charge on the outside of the cube? What is the electric field inside the body of the cubic conductor?
(b) Now consider a Gaussian surface which completely surrounds the cube. What is the total electric flux which goes through that surface?

2. (50 pts) (a) An infinitely long cylindrical charge distribution, $\rho = Cr$ for $r \le a$, is located on the z-axis of a cylindrical coordinate system, where C and a are both constants. Find the electric field both inside and outside the charge distribution. Also find the potential, both inside and outside the charge distribution, assuming that $\varphi(r = 0) = 0$.

(b) Show that your potential satisfies Poisson's equation, $\nabla^2\varphi = -4\pi\rho$, with the appropriate charge density for $r \le a$, and that it satisfies Laplace's equation, $\nabla^2\varphi = 0$, for $r > a$.

3. (20 pts) Derive the capacitance of a concentric spherical capacitor, which has an inner conductor of radius a and an outer conductor of radius b.

(b) Find the energy stored in the electric field between the two spherical shells, and show that it is equal to ½ CV^2.

4. (10 pts) Consider a disc of radius a, located in the xy plane and centered at the origin. In cylindrical coordinates, it has a surface charge $\sigma = Ar$ on its surface, where A is a constant. Find an integral form which describes the potential on the positive z axis. Find an integral form for the electric field on the z-axis. (You do **NOT** need to actually solve the integrals.)

5. (10 pts) Consider the region above a grounded conducting plane with charges of $+Q_1$ and $-Q_2$ located above it as shown. Use the Cartesian coordinate system in which the grounded conducting plane is located in the x-y plane, charge of $+Q_1$ is located at $(0,0,h_1)$, and the charge of $-Q_2$ is located at $(d,0,h_2)$. (a) Write an expression for the potential at any point (x,y,z) in the upper half plane ($z \ge 0$) in the given Cartesian coordinate system.
(b) Explain how you would find the electric field $\vec{E}(x,y,z)$ from the potential. (You do **NOT** need to find an explicit expression for the field from your potential, however.)
(c) Assume that $|Q_2| > |Q_1|$.Draw some typical electric field lines in the upper half plane. Please include some field lines near the charges, as well as some near the plane.