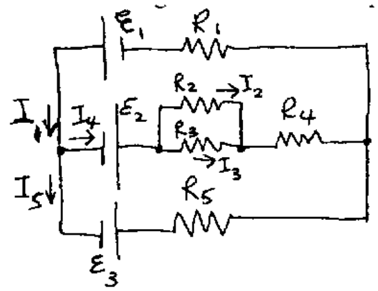
Solutions to Practice Problems for Midterm 2, Physics 9HD

1.



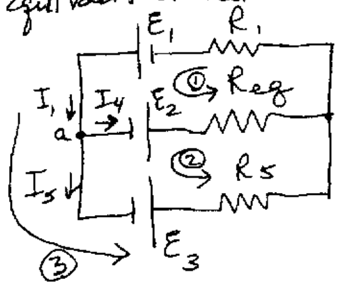
a) $R_{||}$ is parallel equivalent resistance for R_2 + R_3

$$\frac{1}{R_{||}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_3 + R_2}{R_2 R_3}$$

$$R_{||} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{eq} = R_{||} + R_4 = \frac{R_2 R_3}{R_2 + R_3} + R_4$$

Equivalent circuit



b) Currents at node a: $I_1 = I_4 + I_5$

Loop ①: $\mathcal{E}_1 + \mathcal{E}_2 - I_4 R_{eq} - I_1 R_1 = 0$

Loop ②: $-\mathcal{E}_2 + \mathcal{E}_3 - I_5 R_5 + I_4 R_{eq} = 0$

Loop ③: $\mathcal{E}_1 + \mathcal{E}_3 - I_5 R_5 - I_1 R_1 = 0$

Any 3 of the above 4 equations is acceptable.

c) If I_4 is known, V_{eq} = voltage across $R_{eq} = I_4 R_{eq}$

V_4 = voltage across $R_4 = I_4 R_4$

Let V_2 = voltage across R_2

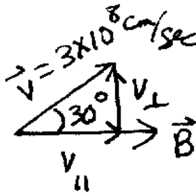
Let V_3 = voltage across R_3

Since R_2 is in parallel with R_3, $V_2 = V_3 = V_{eq} - V_4$

$$I_2 = \frac{V_2}{R_2} = \frac{V_{eq} - V_4}{R_2} = \frac{I_4 (R_{eq} - R_4)}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_{eq} - V_4}{R_3} = \frac{I_4 (R_{eq} - R_4)}{R_3}$$

2.



$$\vec{F} = q\,\frac{\vec{v}}{c}\times\vec{B}$$

$$|\vec{F}| = \frac{(4.8\times10^{-10}\,\text{esu})(3\times10^{8}\,\frac{\text{cm}}{\text{sec}})(5000\,\text{gauss})\sin 30°}{3\times10^{10}\,\frac{\text{cm}}{\text{sec}}}$$

$$|\vec{F}| = 1.2\times10^{-8}\text{ dyne}$$

$\vec{v}\times\vec{B}$ is into paper; Since e^- is negative, $\vec{F}$ points out of paper if $\vec{v}+\vec{B}$ are as shown.

Motion is a helix. $V_\parallel$ along direction of $\vec{B}$ is unchanged.

Redraw diagram with $\vec{B}$ out of paper

$V_\parallel$ out of paper. So helix spirals out of paper counterclockwise if $\vec{B}$ points out of paper

$$m v_\perp c = B q r$$

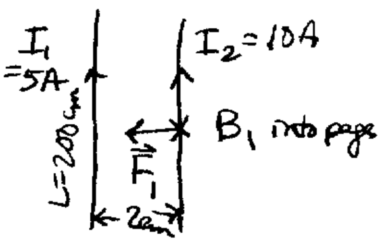
$$\omega = \frac{v}{r} = \frac{Bq}{mc} = \frac{(5000\,\text{gauss})(4.8\times10^{-10}\,\text{esu})}{(9.11\times10^{-28}\,\text{g})(3\times10^{10}\,\frac{\text{cm}}{\text{sec}})} = 8.78\times10^{10}\,\text{sec}^{-1}$$

$$f = \frac{\omega}{2\pi} = 1.4\times10^{10}\text{ Hz} = 14\text{ GHz}$$

3. $$F = \frac{2 I_1 I_2 L}{c^2 r}$$

Since $e = 4.8\times10^{-10}$ esu $= 1.6\times10^{-19}$ coul, $1\text{A}\times\frac{4.8\times10^{-10}\,\text{esu}}{1.6\times10^{-19}\,\text{coul}} = 3\times10^{9}\,\frac{\text{esu}}{\text{sec}}$

$$F = \frac{2(5\text{A})(10\text{A})(3\times10^{9}\,\frac{\text{esu}}{\text{sec}})^2(200\text{ cm})}{(3\times10^{10}\,\frac{\text{cm}}{\text{sec}})^2(2\text{ cm})} = 100\text{ dyne}$$



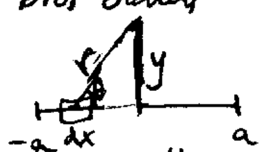
Wires attract if the currents go in the same direction.

Prove this by thinking about $\vec{B}$ field directions + forces.

By right hand rule, B_1 at position of wire 2 goes into page.

$\vec{F}_1 = I\,\vec{l}\times\vec{B}/c$ points to left, i.e. force between 2 wires is attractive, with currents in the same direction, as shown.

4. Biot-Savart $d\vec{B} = \frac{I\,d\vec{l}\times\hat{r}}{cr^2}$. Suppose the wire extends from $-a$ to a along x-axis. Find field at $(0,y,0)$, then take $\lim_{a\to\infty}$ for infinite wire.



$\sin\theta = \frac{y}{r}$

$$\vec{B} = \frac{I}{c}\int_{-a}^{a}\frac{dx}{r^2}\,\frac{y}{r}\,\hat{z} = \frac{I\hat{z}}{c}\int_{-a}^{a}\frac{y\,dx}{(x^2+y^2)^{3/2}} = \frac{I\hat{z}}{c}\left[\frac{x}{y\sqrt{x^2+y^2}}\right]_{-a}^{a}$$

$$\vec{B} = \frac{I\hat{z}}{c}\,\frac{(2a)}{y\sqrt{a^2+y^2}} \xrightarrow{a\gg y} \frac{I\hat{z}}{c}\,\frac{(2a)}{ya} = \boxed{\frac{2I\hat{z}}{cy}}$$

From Ampere's law, $\oint_C \vec{B}\cdot d\vec{s} = 4\pi\frac{I_{encl}}{c} \Rightarrow B(2\pi r) = \frac{4\pi I}{c}$

$\therefore B = \frac{2I}{cr}$, direction is around wire.

If I is along $+x$ axis, B is in $\hat{\varphi}$ direction.

5. $\vec{J} = Ar^2$, $0 \le r \le a$

Ampere's law, $\oint_C \vec{B}\cdot d\vec{s} = \frac{4\pi I_{encl}}{c} = \frac{4\pi}{c}\int_S \vec{J}\cdot d\vec{a}$, where C is curve bounding surface S.

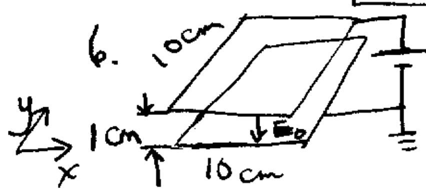
$0 \le r \le a$, Take C to be circle of radius r $\Rightarrow B(2\pi r) = \frac{4\pi}{c}\int_0^r A r'^2 (2\pi r' dr')$

$$B(2\pi r) = \frac{8\pi^2}{c} A \int_0^r r'^3 dr' = \frac{8\pi^2 A}{c}\frac{r^4}{4}$$

$0 \le r \le a$, $\boxed{B = \frac{\pi A}{c} r^3}$

$r \ge a$, C is circle of radius r, $B(2\pi r) = \frac{4\pi}{c}\int_0^a A r'^2 (2\pi r' dr')$

$r \ge a$, $\boxed{B = \frac{\pi A}{c}\frac{a^4}{r}}$

6. 

a) Capacitor charged to 100V = $\frac{1}{3}$ statvolt

$E_0 = \frac{1}{3}\frac{\text{statvolt}}{\text{cm}} = \boxed{0.33 \frac{\text{statvolt}}{\text{cm}}} = 4\pi\sigma_0$ in frame F, $\boxed{\sigma_0 = 2.62\times10^{-2}\ \text{esu}/\text{cm}^2}$

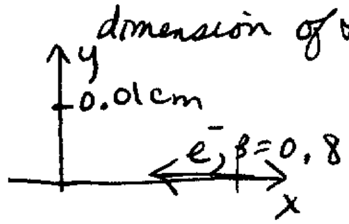
Frame F' moves at speed along x-axis with respect to F.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.6)^2}} = 1.25$$

In F', $E' = \gamma E_0 = 1.25\left(0.33\frac{\text{statvolt}}{\text{cm}}\right) = 4.13\times10^{-1}\frac{\text{statvolt}}{\text{cm}} = \boxed{0.41\frac{\text{statvolt}}{\text{cm}}}$

$$\sigma_0' = \frac{E'}{4\pi} = \boxed{3.28\times10^{-2}\ \text{esu}/\text{cm}^2}$$

b) If capacitor plates are parallel to yz plane, there is no length contraction of either dimension of the plates. ∴ $\boxed{E' = E_0 = 0.33\frac{\text{statvolt}}{\text{cm}},\ \sigma' = \sigma_0 = 0.026\frac{\text{esu}}{\text{cm}^2}}$

7. 

$\vec{E}_{max}$ in lab occurs when e^- is at origin in lab frame.

$\vec{E}_{max} = \frac{e}{\gamma^2 y^2 (1-\beta^2)^{3/2}} = \frac{\gamma e}{y^2}$ from eq. 5.12, with $\sin\theta' = 1$

$$\beta = 0.8,\quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.8)^2}} = 1.67$$

$|\vec{E}_{max}| = \frac{(1.67)(4.8\times10^{-10}\ \text{esu})}{(0.01\ \text{cm})^2} = \boxed{8.0\times10^{-6}\ \frac{\text{esu}}{\text{cm}^2}}$, $\vec{E}_{max}$ is in $-\hat{y}$ direction.

$|F_{proton}| = |eE_{max}| = (4.8\times10^{-10}\ \text{esu})\left(8.0\times10^{-6}\frac{\text{esu}}{\text{cm}^2}\right) = \boxed{3.84\times10^{-15}\ \text{dyne}}$

Force is attractive between e^- + p; ∴ $\vec{F}$ is in $-\hat{y}$ direction.

Field lines of traveling e^-, as viewed from lab frame, look like whiskbroom (or pancake, according to Purcell).