PHYSICS 9HE--SPRING, 2008: STUDY GUIDE TO IMPORTANT IDEAS, EXPERIMENTS, CALCULATIONS

Special Relativity

-Know the postulates of Special Relativity
-Proper time, proper length
-Time dilation (muon decay, planes circling earth), length contraction
-Radiactive decay: as relevant to muon experiment and later discussions of nuclear properties and particle physics
-Doppler shift for light and sound: application to Doppler radar, auto speed monitoring, expansion of universe
-Energy-mass relationships, including reactions in which mass change becomes energy released/taken in (incl. nuclear binding energies from Ch. 12)

General Relativity

-Know the postulate of General Relativity (Principle of Equivalence)
-Effect of gravity on light, including shift of frequency, gravitational lensing & dark matter
-Effect of gravity on time (watch confusion with time dilation in Special Relativity)
-Practical example: GPS satellites for which both Special and General Relativity important
-Black holes and the Schwartzchild radius
-Motion of the perihelion of Mercury
-Gravity waves, the graviton

Waves and Introductory Quantum Ideas

From some basic experiments ala 9HC:
-Emission and absorption spectra
-Blackbody radiation:
Planck's model vs. Rayleigh-Jeans model
Planck Blackbody formula and its meaning
Stefan-Boltzmann formula and Wein displacement laws and their uses
-The photoelectriceffect
-Photon energy and photon momentum (E = pc only for particles of zero rest mass like photon..or other particles at <u>very</u> high energy...so be careful!)
-X-ray production in an x-ray tube:
Via two processes: bremsstrahlung and core electronic transitions
Moseley correlations of x-ray energy with atomic number, electron screening, and effective nuclear charges
Selection rules in x-ray transitions (from later disc. of time dependence in QM)
-Elastic wavelike scattering of x-rays--Bragg diffraction of x-rays by atomic layers in crystals: $n\lambda = 2d\sin\theta$
-Electron-positron pair production, the positronium atom, and positron emission tomography (see discussion at end of quarter)
-Emission and absorption spectra of atoms, including selection rules (from later disc. of time dependence in QM)
-De Broglie wavelength for a particle: $\lambda = h/p$
-Davisson-Germer diffraction of electrons by crystal surfaces near surfaces: 2d and 3d aspects
-The double-slit experiment with light and electrons: building up images by counting
-Conserving momentum and energy in emission of light and creation/annihilation of particles

Basic Concepts and Mathematics of Quantum Mechanics, Including Fourier Transforms

-Complex variables and complex functions—brief review
-Superposition of waves, wave packets, phase and group velocities

-Fourier series and integrals; use of and relationship to the Uncertainty Principles
-Example of RC electrical circuit as a Fourier filter, passing only low frequencies
-Sets of orthogonal, or orthogonal and normalized functions as basis sets
-The Heisenberg Uncertainty Principles in position-momentum
and energy-time
-Wave packets, group and phase velocities
-Know the postulates of Quantum Mechanics and how to use them, including the general rules for solving and understanding a Q.M. problem (see lecture slides)
-The Correspondence Principle
-Time-dependent and time-independent Schroedinger Equations and wave functions: first example of the use of separation of variables
-Other uses of separation of variables in solving time-dependent Sch. Eqn., 3D particle in a rigid box, hydrogenic atom, separation of nuclear and electronic motions in molecules, and other q.m. problems
-Calculation of probability densities and expectation values of operators: make use of even-odd character and symmetry wherever you can to simplify this. E.g.- an overall odd integrand, when integrated over $-\infty < x < +\infty$ must be zero
-Estimation of uncertainties via semi-classical approximations and the Uncertainty Principle
-Eigenfunction/eigenvalue relationships for different wave functions, including
Parity operator $\hat{\Pi}$ defined to keep track of even and odd wavefunctions in potentials that are even:
$\hat{\Pi}\psi(x) = \psi(-x) = \pm 1\psi(x)$, with eigenvalues of +1 (even ψ) and - 1 (odd ψ)
(E.g., see table on 1D problems in lecture slide)

Quantum Mechanics in 1 Dimension

-Types of solutions and boundary conditions arising for different potentials:
Particle in a rigid box
Particle in a soft box
Harmonic oscillator and application to molecular vibrations
...including Correspondence Principle limits
-Extension of rigid box to 3D by separation of variables:
Degeneracy = several independent ψ's yield same energy
-Be able to sketch qualitative form of a wave function in an arbitrary potential, assuming that it is piecewise constant, and using wavevector (k) or decay constant (κ) and Correspondence Principle arguments (e.g., $|\psi|^2$ is maximum where classical particle moves most slowly)
-Tunneling effects: general form of wavefunctions in different regions and calculation of tunneling probability T(E) for special cases: high-wide barrier, alpha decay, tunnel diode, field emission, and scanning tunneling microscope (see lecture slides)

Hydrogenic Atom Wave Functions and Properties

-Separable form in r,θ, and ϕ: $\psi_{n\ell m_\ell}(r,\theta,\phi) = R_{n\ell}(r)\Theta_{\ell m_\ell}(\theta)\frac{1}{\sqrt{2\pi}}e^{im_\ell\phi}$

-Quantum nos. n, ℓ, and m_ℓ and their significance to energy, angular momentum, and parity eigenvalues:

$$\textit{Energy}: \quad \hat{H}\psi_{n\ell m_\ell} = E_n\psi_{n\ell m_\ell}, E_n \text{ from Bohr formula} = -\frac{Z^2e^2}{8\pi\varepsilon_0 a_0}\frac{1}{n^2}$$

$$\textit{Square of orbital angular momentum}: \quad \hat{L}^2\psi_{n\ell m_\ell} = \hbar^2\ell(\ell+1)\psi_{n\ell m_\ell}$$

$$\textit{Z component of orbital angular momentum}: \hat{L}_z\psi_{n\ell m_\ell} = \hbar m_\ell\psi_{n\ell m_\ell}$$

$$\textit{Parity}: \hat{\Pi}\psi_{n\ell m_\ell}(r,\theta,\phi) = \psi_{n\ell m_\ell}(r,\pi-\theta,\phi+\pi) = (-1)^\ell\,\psi_{n\ell m_\ell}$$

-Normalization and calculation of probabilities and expectation values, including integration with proper volume element in spherical polar coordinates: $dV = r^2 dr \sin\theta d\theta d\phi$

-Use of radial probability distribution $P_{n\ell}(r) = \left[R_{n\ell}(r)\right]^2 r^2$, shell structure associated with a given n value

-Conversion of complex functions for $m_\ell \neq 0$ into the real functions of chemistry via sum and difference

-Graphical representation of different aspects of wave functions in 1D, 2D,and 3D, including radial and angular nodes

-Degeneracy of atomic ψ's

-Vector model of quantization of space for orbital angular momentum

-Magnetic moment associated with orbital angular momentum

-Normal Zeeman effect: different degenerate m_ℓ levels split by magnetic field

-Selection rules for dipole-radiation transitions and application to atomic spectra, x-ray spectra, and molecular rotational and vibrational excitations

Spin Angular Momentum, Spin Magnetic Moment

-Stern-Gerlach experiment and existence of spin $\vec{s}$: one last relativistic effect!

-Adds quantum no. $m_s = \pm 1/2$ and spin magnetic moment $\vec{\mu}_s = -\frac{e}{m}\vec{s}$

-Additional eigenvalue properties with spin added are:

$$\text{Square of spin angular momentum}: \hat{S}^2 \psi_{n\ell m_\ell m_s} = \hbar^2 \frac{1}{2}\left(\frac{1}{2}+1\right)\psi_{n\ell m_\ell m_s} = \frac{3}{4}\hbar^2 \psi_{n\ell m_\ell m_s}$$

$$\text{Z component of spin angular momentum}: \hat{S}_z \psi_{n\ell m_\ell m_s} = \hbar m_s \psi_{n\ell m_\ell m_s} = \pm\frac{\hbar}{2}\psi_{n\ell m_\ell m_s}$$

-Vector model of quantization of space for spin angular momentum

-Spin-orbit coupling and vector model of quantization of space for total angular momentum $\vec{j} = \vec{\ell} + \vec{s}$

Wave Functions and Energies for Multielectron Atoms

-The essential points:

-the electron has intrinsic spin: s = 1/2, and z component linked to $m_s = \pm 1/2$

-this leads to four total quantum nos. for non-relavistic electrons in atoms: n, ℓ, m_ℓ, m_s

-the wave function for a set of identical particles cannot yield results which depend on our choice of particle labelling: e.g., probability density:

$$\left|\psi(\ldots\vec{r}_1\ldots\vec{r}_2\ldots)\right|^2 = \left|\hat{P}_{12}\psi(\ldots\vec{r}_1\ldots\vec{r}_2\ldots)\right|^2 = \left|\psi(\ldots\vec{r}_2\ldots\vec{r}_1\ldots)\right|^2,$$

where $\hat{P}_{12}$ is the permutation operator which just interchanges the labels of all space and spin coordinates, thus "trading the places" of electrons 1 and 2 in ψ.

-all particles are "fermions" or "bosons", depending on sign change when any two labels are interchanged. I.e., all valid many-particle wave functions are eigenfunctions of $\hat{P}_{12}$, and all particles are in two groups:

Fermions (electrons and all particles with half-integral spin = 1/2, 3/2, 5/2,...):antisymmetric ψ's with -1 eigenvalue: $\hat{P}_{12}\psi = -1\psi$.

Bosons (photons, all particles with integral spin = 0, 1, 2, 3,...): symmetric ψ's with +1 eigenvalue: $\hat{P}_{12}\psi = +1\psi$.

-This leads to the Pauli Exclusion Principle for electrons (or all Fermions):

No two electrons in a multielectron atom can have all of n, ℓ,m_ℓ,m_s (or n, l, j, and m_j) the same. (If they do, then ψ is trivially zero!)

- And also: Electrons with the same spin orientation (up, up) or (down, down) are never found at the same point in space, lowering their Coulomb repulsion ("exchange interaction"): explains magnetism and Hund's First Ruld.

-Inner-shell electron screening in multi-electron atoms lifts the H-atom degeneracy in ℓ through an effective Z that varies with ℓ for a given n: energies always go as s < p < d < f, with overlap of n values which e.g. makes 4s fill before 3d

-Pauli Principle then leads to filling of atomic levels in Periodic Table. If open shell at the end, Hund's First Rule says fill with maximum total spin S. (Don't worry about other Rules.)

Molecules

-"Electrons in a box" with one potential well on each atom; for diatomics--wave functions approximated as sums (bonding) and differences (anti-bonding) of atomic functions on each atom
-For polyatomic molecules, electronic wave functions describable as linear combinations of atomic orbitals on different atoms
-Diatomics also behave like 1D harmonic oscillators in their vibrations
-Diatomics also behave like rigid rotors in their rotations
-Examples of CO microwave absorption and ultraviolet photoelectron excitation, and the microwave oven

Maxwell Boltzmann, Fermi-Dirac and Bose-Einstein Statistics (Review)

-MB, FD, and BE statistics, basic formulas, similarities and differences
-Blackbody radiation as BE example, including solar spectrum, Cosmic microwave backgnd.
-Electrons as FD example (see below in solids)

Lasers and Holography

-General principles of stimulated emission and population inversion (qualitative)
-Coherence (high monochromaticity) of laser light and application to holography (qualitative)
-Example of photoelectron holography

Solids and Solid State Devices

-Electron counting and level filling in solids
-Free-electron-like solids: density of states, Fermi energy, Fermi wave vector and velocity
-Li and Al as examples of nearly free-electron solids
-Periodic solids and Bloch functions as "universal" wave function form for solids
-The Kronig-Penney model and energy bands
-Semiconductor energy bands—e.g. Ge
-Metallic energy bands—e.g. non-magnetic Cu and ferromagnetic Fe
-Phenomenology of ferromagnetism and superconductivity/Cooper pairs
-Ionic solids and the Madelung sum
-Semiconductor doping of n or p type
-The p-n junction diode, extension to the LED and the photovoltaic cell
-The metal-oxide-semiconductor field-effect transistor (MOSFET)
-Nanoscience and nanotechnology
-Moore's Law for integrated circuits; analogue for magnetic bit density in storage media

Nuclear Properties and Applications

-Decay of nuclei and particles and half lives

-Notation including Z, N, and A and chemical symbol: ${}^{A}_{Z}X_{N}$

-Nuclear sizes, charge density versus radius, Rutherford scattering (qualitative)

-Nuclear binding energies per nucleon, calculation of and stability versus mass number A

-The nuclear interactions: proton-proton Coulomb barrier plus strong nucleon-nucleon Interaction plus hard-core repulsion

-Shell model (may give net spin on nucleus) and liquid drop model for calculating binding energies per nucleon

-Five basic decay modes and meaning for changes in ${}^{A}_{Z}X_{N}$

-Applications to carbon dating, nuclear energy production by fission and fusion, understanding radiation from radon in earth's crust

-Enrichment of U by diffusion and centriguation, breeder reactors using Pu

-Nuclear spin, magnetic resonance (qualitative), Magnetic Resonance Imaging (MRI) in comparison to Positron Emission Tomography (PET)

Elementary Particles and Interactions

-Particles, antiparticles

-Leptons and Quarks, the table of elementary particles in the Standard Model and the manner in which quarks go together to make up common Hadrons = mesons (2 quarks) + baryons (3 quarks, e.g., protons and neutrons)

-Four fundamental forces: electromagnetic, weak, strong, and gravitational, and four Bosons mediating them (photon; W,Z bosons; gluons; and gravitons, respectively)

-Dark matter (e.g. from gravitational lensing) and dark energy (e.g. from increase in rate of universe expansion)