Physics 115a
Homework 3 (due April 19 at start of class)
Assigned April 13
Corrected and clarified April 15 9am
Additional clarifications added in red April 16 9:45 pm

3.1) Take the size of the observed universe to be $R_U = 10^{25} m$. Consider a particle with mass m in a one dimensional infinite square well potential with width equal to R_U.

a) Write down an expression for the n-th energy eigenvalue of this system. Please give your answer in the form

$$E_n = f(n) E_1 \tag{1.1}$$

b) Evaluate E_1 in your answer to a) for the following masses:

i) Electron Mass
ii) Human mass (100kg)
iv) Mass of the earth

c) Consider

$$\Delta_n \equiv \frac{E_{n+1} - E_n}{E_n} \tag{1.2}$$

i) For what value of n is Δ_n maximized?

ii) Evaluate Δ_n for each of the objects considered in homework problem 2.4 a, b and c. In each case set E_n equal to the kinetic energy to find n. *(You need calculate the value of n to one or two digits).*

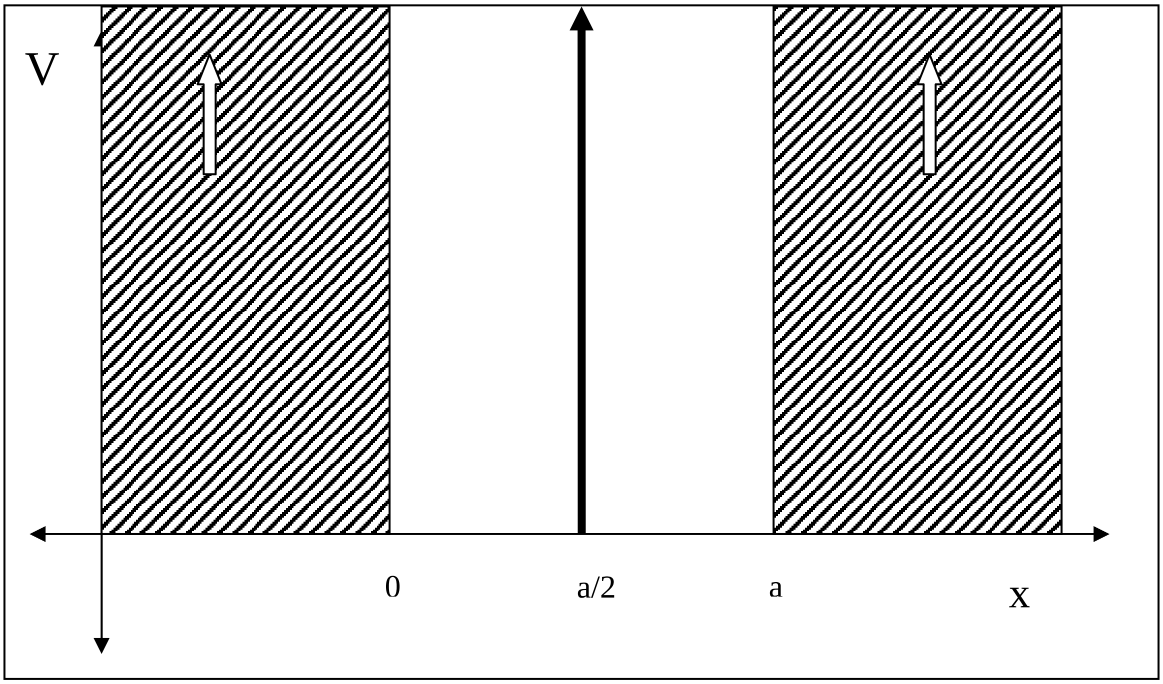
d) For each energy eigenstate one can construct the classical momentum p_n assuming all E_n is kinetic energy. (A reasonable choice for the square well). Construct the general form for

$$\Delta p_n \equiv \frac{p_{n+1} - p_n}{p_n} \tag{1.3}$$

for this system and then evaluate it for the three cases consider in part c) of this problem.

Think about: *If we really were living inside a square well of size R_U, do you think it would be easy to tell the difference between that and a continuum (which would allow a continuous range of values for energy and momentum)? This is for you to think about, but I'm not asking for a response written in this homework.*

3.2) Consider the variation on the infinite square well which I depict in the figure: The square well has width a but it also has an infinitely high barrier of zero thickness at $x = a/2$. The effect of this barrier is to force the same boundary conditions on the wave function at the barrier (on either side) as apply on the edges of the square well.



a) Now consider the energy eigenstates of the simple square well *without* the barrier at $x = a/2$. Some of these are also energy eigenstates of the square well *with* the barrier at the center and some are not. Which normal square well eigenstates are eigenstates of both potentials, and which ones are not? You must explain your reasoning to get credit for this question.

b) Which eigenstate of the potential with the central barrier has the lowest energy?

c) *By comparison of the boundary conditions and the eigenvalue equations which apply inside the well, one can conclude that any eigenstate of the well with the central barrier must also be an eigenstate of an infinite square well of width $a/2$ when viewed separately in the x ranges $0 < x < a/2$ and $a/2 < x < a$. (You need not prove the previous statement. I am asserting that it is obvious.) Note that you will not get a normalized eigenstate this way (until you re-scale the constant factor in front), but it is an eigenstate none the less.* Which energy eigenstate(s) of the half-wide well correspond to the left and right sides of the eigenstate you gave in your answer to part b) of this problem?

d) Call the wavefunction for state in your answer to part b) ψ_1. (This change of notation may not be ideal, but it will help with what follows.) Now construct another wavefunction (called ψ_{-1}) by flipping the sign of $\psi_1(x)$ *only* for $0 < x < a/2$. That is

$$\psi_{-1}(x) = \begin{cases} -\psi_1(x) & 0 < x < a/2 \\ \psi_1(x) & a/2 < x < a \end{cases} \tag{1.4}$$

Is ψ_{-1} an energy eigenstate? If so, give its energy eigenvalue.

e) Answer part c) for ψ_{-1}

f) Evaluate $\int_{-\infty}^{\infty} \psi_{-1}^*(x)\psi_1(x)\,dx$. Are ψ_{-1} and ψ_1 orthogonal? *You can relate the value of the inner product to the orthogonality of the vectors the same way you would relate them for familiar vectors in 3d space.*

g) Use the earlier sections of this problem to infer the full spectrum of energy eigenstates (and their corresponding eigenvalues) for the well with the central barrier. Please use the convention that (without loss of generality) all the energy eigenstate wavefunctions can be taken to be real.

h) Compare the number of energy eigenstates within any specified energy range for these two cases: i) The plain infinite square well and ii) the infinite square well with the central barrier depicted in this problem.