

Physics 115a

Homework 6 (due May 17 at start of class)

Assigned May 10

Updated May 13

6.1) Consider the double delta function potential given by:

$$V(x) = -\alpha\delta(x+a) - \alpha\delta(x-a) \quad (1.1)$$

a) Find an even bound state wavefunction. Give the functional form of the wavefunction and the energy (in terms of parameters of the potential and the particle mass).

b) Sketch the function you found in part a)

c) Find an odd bound state wavefunction. Give the functional form of the wavefunction and the energy (in terms of parameters of the potential and the particle mass).

d) Sketch the function you found in part c)

Hint: You may assume the evenness and oddness from the start, as you set out to do parts a) and c) respectively.

Update added May 13: *The general solutions you get for both wavefunction give only implicit equations for $\kappa(a, \alpha)$. please give only those implicit equations as your answers above. Then analyze the implicit equations using part e) below. ALSO, I am just looking for rough features in the sketches above. (i.e decaying vs oscillating solutions, breaks in the slope, correct behavior at large distances, evenness or oddness). If you want something more concrete so you can plot a specific solution you can use the small $\tilde{\kappa}$ limit discussed below to come up with a concrete example*

e)

i) You can simplify your implicit equations for $\kappa(a, \alpha)$ slightly by writing them in terms of the dimensionless wavenumber $\tilde{\kappa} \equiv \kappa a$ and $b \equiv \frac{2m\alpha a}{\hbar^2}$, a dimensionless measure of the potential strength. Make that simplification.

ii) Solve for $\tilde{\kappa}(b)$ for each case (even/odd) in the limit of small $\tilde{\kappa}$.

iii) For which case(s) (even/odd) does an energy eigenstates exist for b ?