1. $\gamma_v \geq 1$. Classical mechanics applies when $v \ll c$, and $\gamma_v = 1$. At what speed will γ_v be 1.01?

$$\frac{1}{\sqrt{1-v^2/c^2}} = 1.01 \Rightarrow \mathbf{0.14c}$$

2. Your time is longer. $\Delta t_{you} = \gamma_v \, \Delta t_{Carl} \rightarrow 60s = \frac{1}{\sqrt{1-(0.5)^2}} \Delta t_{Carl} \Rightarrow \Delta t_{Carl} = \mathbf{52s}$

3. $L = L_o \sqrt{1-v^2/c^2} \rightarrow 35m = L_o \sqrt{1-(0.6)^2} \Rightarrow L_o = \mathbf{43.75m}$

10. Since Bob moves to the right, let's make his frame S´. Thus, we seek times in frame S. $\gamma_v = 2$ $\rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow v = \sqrt{3}/2\ c$. The clock at the back of Anna's ship is presently at $x' = \frac{1}{2} L_o$. (It is also at $x = L_o$.) Bob looks at this clock at $t' = 0$ on his own clock. Call this Event a. With $x_a' = \frac{1}{2} L_o$, $t_a' = 0$, we may find t_a, the time in Anna's frame.

$t_a = \gamma_v \left(+\frac{v}{c^2} x_a' + t_a'\right) = 2\left(\frac{\sqrt{3}/2}{c} \frac{1}{2} L_o + 0\right) = \mathbf{\frac{\sqrt{3}}{2c} L_o}$. The clock at the center of Anna's ship is at $x' = {}^1/_4 L_o$ (rather than $\frac{1}{2} L_o$) in Bob's frame.

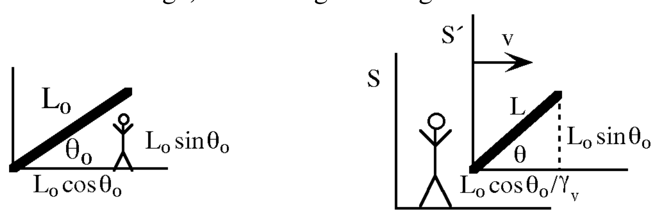
The time on this clock, t_b, will therefore simply be half the previous result. $t_b = \mathbf{\frac{\sqrt{3}}{4c} L_o}$.

13. According to those on the ground, this is simple classical mechanics: a speed and distance according to a ground observer are used to find a time according to a ground observer. Let's call the ground frame S. $\Delta t = \frac{4\times10^6 m}{0.8\times3\times10^8 m/s} = 1.67\times10^{-2}s$. There are two ways to find the time on the spaceplane––time dilation, or length contraction. *Time dilation*: The ground observer will see more time pass on his clock than on the plane. His is Δt, the plane's $\Delta t'$.
(Note: $\gamma_{0.8c} = 1.67$) $\Delta t = \gamma_v \, \Delta t' \rightarrow 1.67\times10^{-2}s = (1.67)\, \Delta t' \Rightarrow \Delta t' = 0.0100s$.
Length contraction: The plane observer sees a different coast–to–coast distance. He sees $L = \sqrt{1-(0.8)^2}\,(4\times10^6 m) = 2.4\times10^6 m$. How much time will it take a 2,400m object to pass at 0.8c? Classical mechanics: $\Delta t' = \frac{2.4\times10^6 m}{0.8\times3\times10^8 m/s} = 0.0100s$. The clock seen on the plane from the ground will be 0.0167s – 0.0100s = **0.0067s behind**. But how can it be behind if the people on the plane must see time on *ground* clocks passing more slowly? Should they not see the *ground* clocks behind? Assuming the plane and ground clocks read zero as the plane started across the country, the plane clock will indeed be behind the ground clock on the other coast, just as we calculated. The people on the plane will *by no means* agree that the clock on the *far* coast was synchronized, that it read zero when the plane started across country (relative simultaneity). In fact, they will say that it was significantly ahead, so that, even though they "see" less time pass on this far–off clock, they will agree that it is ahead when they pass it.

14. We may "work in either frame". *Muon's frame*: $\tau = 2.2\mu s$. Distance to Earth is shorter: $4km \sqrt{1-(0.93)^2} = 1.47km$. $t = \frac{dist}{speed} = \frac{1{,}470m}{0.93\times3\times10^8 m/s} = 5.27\times10^{-6}s$. $\frac{N_o}{N} = e^{-(5.27/2.2)} = e^{-2.4}$ or **9.1%**. *Earth frame*: Distance to Earth is 4km. Lifetime is longer. $\frac{2.2\mu s}{\sqrt{1-(0.93)^2}} = 5.99\mu s$.
$t = \frac{dist}{speed} = \frac{4{,}000m}{0.93\times3\times10^8 m/s} = 1.43\times10^{-5}s$. $\frac{N_o}{N} = e^{-(5.99/14.3)} = e^{-2.4}$ or 9.1%
(b) $\tau = 2.2\mu s$. $t = \frac{dist}{speed} = \frac{4{,}000m}{0.93\times3\times10^8 m/s} = 1.43\times10^{-5}s$. $\frac{N_o}{N} = e^{-(14.3/2.2)} = e^{-6.5}$ or **0.14%**

15. According to observers on the plane, these two events occur at the same location. Their times differ by Δt_o. $\Delta t = \frac{\Delta t_o}{\sqrt{1-v^2/c^2}} \rightarrow \Delta t_o = \left(1-(420/3\times10^8)^2\right)^{½}$ (10s) $\cong \left(1-½\,(420/3\times10^8)^2\right)$ (10s) $= 10\text{s} - 9.8\times10^{-12}\text{s}$. The plane's clock will read **9.8ps earlier**. May also solve in plane frame, where 420m is contracted.

18. The plank has moved to a different frame, frame S´. The length along the x´–axis is $L_x{}' = L_o \cos\theta_o$, and along the y´–axis is $L_y{}' = L_o \sin\theta_o$. An observer in S will see the x–leg contracted, but not the y. $L_x = L_x{}'/\gamma_v = L_o \cos\theta_o \sqrt{1-v^2/c^2}$, and $L_y = L_y{}' = L_o \sin\theta_o$.
$L = \sqrt{L_x{}^2+L_y{}^2} = \sqrt{L_o{}^2\cos^2\theta_o(1-v^2/c^2)+L_o{}^2\sin^2\theta_o}$ .But $\sin^2\theta_o = 1-\cos^2\theta_o$.
$L = L_o\sqrt{\cos^2\theta_o(1-v^2/c^2)+(1-\cos^2\theta_o)} = L_o\sqrt{1-(v^2/c^2)\cos^2\theta_o}$. As the angle θ_o goes from zero to 90°, the length of the plank to an observer in S goes from $L_o\sqrt{1-v^2/c^2}$ (simple length contraction) to merely L_o (no contraction perpendicular to axes of relative motion).
Now, $\tan\theta = \frac{L_y}{L_x} = \frac{L_o\sin\theta_o}{L_o\cos\theta_o\sqrt{1-v^2/c^2}} = \tan\theta_o \frac{1}{\sqrt{1-v^2/c^2}} = \gamma\tan\theta_o$. As the speed increases, the angle θ seen by an observer in S increases. This makes sense in view of the fact that the faster the relative motion, the shorter the x–component of the plank will be.
The y–component doesn't change, so the angle is larger.



23. Calling to the right positive and Anna frame S´, we know that this tick of the clock/event occurs at $t=0$ and $x = ½\ell$ according to Bob. We seek t´. $t' = \gamma_v\left(-\frac{v}{c^2}x+t\right)$. $\gamma_v = \frac{1}{\sqrt{1-\frac{1}{(\sqrt{2})^2}}} = \sqrt{2}$.

Thus $t' = \sqrt{2}\left(-\frac{1/\sqrt{2}}{c}(30\text{m})+0\right) = -\frac{30\text{m}}{3\times10^8\text{m/s}} = -10^{-7}\text{s} =$ **−100ns**. (b) We have a distance traveled according to Bob, 30m, and a speed, and so find a time according to Bob via classical mechanics. $t = \frac{\text{dist.}}{\text{speed}} = \frac{30\text{m}}{c/\sqrt{2}} = \frac{30\text{m}\sqrt{2}}{3\times10^8\text{m/s}} = 1.41\times10^{-7}\text{s} =$ **141ns**. (c) If Anna's back clock has moved to the middle of Bob's ship, Anna's front clock, formerly at Bob's middle, will now be at Bob's front, x=60m. Thus the locations, according to Bob, are x=30 and x=60m. The times are both 141ns. For Anna's back clock: $t' = \sqrt{2}\left(-\frac{1/\sqrt{2}}{c}(30\text{m})+141\times10^{-9}\text{s}\right)$
$= -\frac{30\text{m}}{3\times10^8\text{m/s}} + \sqrt{2}\times141\times10^{-9}\text{s} =$ **100ns**. For Anna's front: $t' = \sqrt{2}\left(-\frac{1/\sqrt{2}}{c}(60\text{m})+141\times10^{-9}\text{s}\right)$
$= -\frac{60\text{m}}{3\times10^8\text{m/s}} + \sqrt{2}\times141\times10^{-9}\text{s} =$ **0**.

(d) In the diagrams, Event 1 shows an observer in Anna's ship, at $t'=-100$ns by her wristwatch, looking at a clock at the middle of Bob's, which reads zero. Event 2 shows another observer in Anna's ship, at $t'=+100$ns by her wristwatch, looking at *the same clock* in Bob's, which reads 141ns. The time elapsing between these events in Anna's frame is $+100-(-100) = 200$ns, but they see this single clock at a fixed location in Bob's ship mark off only 141ns, which is less than 200ns by the usual factor: γ_v. That is, $200\text{ns} = \sqrt{2} \times 141\text{ns}$. Events 3 and 4 show observers in Anna's frame, who are less than 60m apart (not both being at the very ends of their own ship) viewing *at the same time* ($t'=0$) the very ends of Bob's ship. It is less than 60m long according to them. How much less? According to Bob, the two clocks on Anna's ship are 30m apart, but this is a length–contracted observation. They must therefore be more than 30m apart in their own frame: $\sqrt{2} \times 30\text{m} = 42.4\text{m}$. As noted, this is how long Bob's ship appears to observers in Anna's frame. This fits! They should see the length of Bob's ship as $\frac{60\text{m}}{\gamma_v} = \frac{60\text{m}}{\sqrt{2}} = \sqrt{2} \times 30\text{m} = 42.4\text{m}$.

27. The whole idea here is that you are jumping into a new frame, and the clocks in the now moving Earth–Centaurus A frame are unsynchronized. How out of synch is the one on Centaurus A? Let us call the Earth–Centaurus A system frame S, with their separation given the symbol W. You are initially in frame S, but at the passing of the origins you instantly jump into frame S´, moving at 5m/s toward Centaurus A. We know that your time is $t'=0$, and we seek t, the time on a clock in Centaurus A at $x= +W$ in frame S. $t'=\gamma_v\left(t-\frac{v}{c^2}x\right) \rightarrow 0 = \gamma_v\left(t-\frac{v}{c^2}W\right)$, or $t = \frac{v}{c^2}W$. This being positive, the clocks in front of you will all be ahead. $\frac{v}{c^2} = \frac{5\text{m/s}}{9\times10^{16}\text{m}^2/\text{s}^2} = 5.56\times10^{-17}\text{s/m}$. Thus, $t = W\times 5.56\times10^{-17}\text{s/m}$ (a) If $W = 2\times10^{23}$m then clock **jumps ahead** by $t = 1.11\times10^{7}$s = **128days**. (b) If $W = 4.5\times10^{9}$m then t = **250ns**. (c) Need only reverse the sign of x. Clocks will be **behind by same amounts**. (d) If the traveler is moving away from Earth, then decelerates to a stop (*stopping* being the reverse of *starting* to jog *away* from a planet), he moves to a frame in which clocks back on Earth are immediately advanced. If he furthermore turns around and jumps to a frame moving back toward Earth, he moves to a frame in which clocks back on Earth are again immediately advanced. Acceleration toward Earth causes clocks there to advance. The effect depends not only on the speed involved, but also on the distance away; as we see, by merely jogging this way and that at ordinary speeds, we move through frames in which clocks on heavenly bodies *very* far away change by a great deal. Readings on local clocks, those around the solar system, don't vary much.