1. $KE_{max} = hf - \phi \rightarrow \frac{1}{2}(9.11\times10^{-31}\text{kg})(0.002\times3\times10^{8}\text{m/s})^2 = (6.63\times10^{-34}\text{J}\cdot\text{s})\left(\frac{3\times10^{8}\text{m/s}}{300\times10^{-9}\text{m}}\right) - \phi$

(The classical expression for KE is OK since $\frac{v}{c} \ll 1$.) $\phi = 4.99\times10^{-19}\text{J} =$ **3.12eV**. (b) The cutoff wavelength is the longest (smallest f) that can eject electrons––no KE to spare. $KE_{max} = hf - \phi$ $\rightarrow 0 = (6.63\times10^{-34}\text{J}\cdot\text{s})\, f - 4.99\times10^{-19}\text{J} \Rightarrow f = 7.53\times10^{14}\text{Hz}$. $\lambda = \frac{3\times10^{8}}{7.53\times10^{14}\text{Hz}} =$ **399nm**

2. $KE_{max} = hf - \phi = (6.63\times10^{-34}\text{J}\cdot\text{s})\left(\frac{3\times10^{8}\text{m/s}}{250\times10^{-9}\text{m}}\right)(6.25\times10^{18}\text{eV/J}) - 4.3\text{eV} = 0.67\text{eV}$. **0.67V**

3. $KE_{max} = hf - \phi \rightarrow \frac{1}{2}(9.11\times10^{-31}\text{kg})(2\times10^{6}\text{m/s})^2$
$= (6.63\times10^{-34}\text{J}\cdot\text{s})\left(\frac{3\times10^{8}\text{m/s}}{\lambda}\right) - 3.7\times1.6\times10^{-19}\text{J} \Rightarrow \lambda =$ **82.4nm**

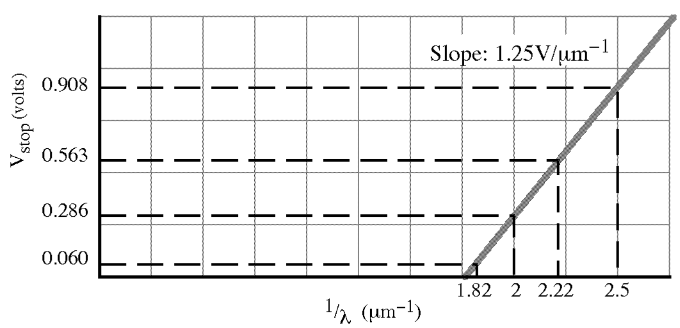
4. $\frac{\text{photons}}{\text{sec}} = \frac{\text{energy/sec}}{\text{energy/photon}} \rightarrow 6\times10^{15}\frac{\text{photons}}{\text{sec}} = \frac{0.002\text{J/s}}{(6.63\times10^{-34}\text{J}\cdot\text{s})\left(\frac{3\times10^{8}\text{m/s}}{\lambda}\right)} \Rightarrow \lambda =$ **597nm**

5. $\frac{\text{energy/time}}{\text{energy/photon}} = \frac{40\times10^{3}\text{J/s}}{(6.63\times10^{-34}\text{J}\cdot\text{s})(940\times10^{3}\text{Hz})} =$ **6.42×10^{31} photons per sec**

6. $E = h\, c/\lambda \geq 1.2\text{eV} \rightarrow (6.63\times10^{-34}\text{J}\cdot\text{s})\left(\frac{3\times10^{8}\text{m/s}}{\lambda}\right) \geq 1.2\times1.6\times10^{-19}\text{J} \Rightarrow$ **$\lambda \leq 1{,}036$nm**
All visible light (~400–700nm) would thus be capable of exposing film.

8. $\frac{1}{2}(9.11\times10^{-31}\text{kg})(1.78\times10^{5}\text{m/s})^2 = (6.63\times10^{-34}\text{J}\cdot\text{s})\left(\frac{3\times10^{8}\text{m/s}}{520\times10^{-9}\text{m}}\right) - \phi \Rightarrow \phi = 3.68\times10^{-19}\text{J} = 2.3\text{eV}$.
$\frac{1}{2}(9.11\times10^{-31}\text{kg})(4.81\times10^{5}\text{m/s})^2 = (6.63\times10^{-34}\text{J}\cdot\text{s})\left(\frac{3\times10^{8}\text{m/s}}{\lambda}\right) - 3.68\times10^{-19}\text{J} \Rightarrow \lambda=$ **420nm**
(b) It would seem to match **sodium**.

9. $KE_{max} = hf - \phi \rightarrow e\cdot V_{stop} = h\frac{c}{\lambda} - \phi$. Could plug in V_{stop} values and λ values, yielding three equations in two unknowns (h and ϕ). Better: graph V_{stop} vs. $1/\lambda$, which should be a linear plot, of slope $h\frac{c}{e}$. From plot, slope is $1.25\text{V}/\mu\text{m}^{-1}$. $1.25\times10^{-6}\text{V}\cdot\text{m} = h\,\frac{3\times10^{8}\text{m/s}}{1.6\times10^{-19}\text{C}}$
$\Rightarrow h = 6.67\times10^{-34}\text{J}\cdot\text{s}$. Correct within less than 1%.



11. $KE_{electron} \rightarrow h\frac{c}{\lambda}$. Thus $\frac{1}{2}(9.11\times10^{-31}\text{kg})\,v^2 = (6.63\times10^{-34}\text{J}\cdot\text{s})\,\frac{3\times10^{8}\text{m/s}}{6.2\times10^{-11}\text{m}}$
$\Rightarrow v = 8.4\times10^{7}\text{m/s}$. This is pretty fast. Let's try the relativistic formula:
$(\gamma_u - 1)(9.11\times10^{-31}\text{kg})(3\times10^{8}\text{m/s})^2 = (6.63\times10^{-34}\text{J}\cdot\text{s})\,\frac{3\times10^{8}\text{m/s}}{6.2\times10^{-11}\text{m}} \Rightarrow u = 0.272c =$ **8.15×10^{7}m/s**

12. $30\text{keV} = h\frac{c}{\lambda} \Rightarrow \lambda = \frac{(6.63\times10^{-34}\text{J·s})(3\times10^{8}\text{m/s})}{30\times10^{3}\times1.6\times10^{-19}\text{J}}$ = **0.0414nm**.

13. $KE_f - KE_i = 2\times(\gamma_{0.6c}-1)m_d c^2 - h\frac{c}{\lambda} \rightarrow$

$2(\frac{5}{4}-1)(2.01355\text{u}\times1.66\times10^{-27}\text{kg/u})(9\times10^{16}\text{m}^2/\text{s}^2) - (6.63\times10^{-34}\text{J·s})\frac{3\times10^{8}\text{m/s}}{1.29\times10^{-15}\text{m}} = -3.8^{-12}\text{J}.$

$-\Delta mc^2 = -(2\times(2.01355\text{u}) - 4.00151)\times1.66\times10^{-27}\text{kg}(9\times10^{16}\text{m}^2/\text{s}^2) = -3.8\times10^{-12}\text{J}$

14. A 10eV photon would have λ given by: $E = h\,{}^{c}/_{\lambda} \rightarrow 10\times1.6\times10^{-19}\text{J} = 6.63\times10^{-34}\text{J·s}\,\frac{3\times10^{8}\text{m/s}}{\lambda}$ $\Rightarrow \lambda$ = 124nm. X–rays have wavelengths many orders of magnitude smaller, so would have energies many orders of magnitude larger. The 10eV could be ignored

16. The fastest are those that are hit head–on, such that $\theta=180°$. $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

$\rightarrow \lambda' - 0.065\times10^{-9}\text{m} = \frac{6.63\times10^{-34}\text{J·s}}{(9.11\times10^{-31}\text{kg})(3\times10^{8}\text{m/s})}(1-(-1)) \Rightarrow \lambda' = 6.99\times10^{-11}\text{m}.$

The electron's kinetic energy is the difference between the photon energies.

$KE_e = (\gamma_u - 1)m_e c^2 = h\,{}^{c}/_{\lambda} - h\,{}^{c}/_{\lambda'} \rightarrow (\gamma_u-1)(9.11\times10^{-31}\text{kg})(9\times10^{16}\text{m}^2/\text{s}^2) =$

$(6.63\times10^{-34}\text{J·s})(3\times10^{8}\text{m/s})\left(\frac{1}{6.5\times10^{-11}\text{m}} - \frac{1}{6.99\times10^{-11}\text{m}}\right) \Rightarrow u = 0.0719c =$ **2.16×10^{7}m/s**

17. $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \rightarrow 0.061\times10^{-9}\text{m} - 0.057\times10^{-9}\text{m} = \frac{6.63\times10^{-34}\text{J·s}}{(9.11\times10^{-31}\text{kg})(3\times10^{8}\text{m/s})}(1-\cos\theta)$

$\Rightarrow \theta =$ **130.5°**. (b) Here's one of many ways: Using (2–3a), $\frac{h}{\lambda} = \frac{h}{\lambda'}\cos 130.5° + \gamma_u m_e u\cos\phi$

$\rightarrow \frac{h}{0.057\text{nm}} - \frac{h}{0.061\text{nm}}\cos 130.5° = \gamma_u m_e u\cos\phi$. Using (2–3b), $\frac{h}{0.061\text{nm}}\sin 130.5° =$

$\gamma_u m_e u \sin\phi$. Dividing second by first, $\frac{\sin 130.5°/0.061}{1/0.057 - \cos 130.5°/0.061} = \tan\phi \Rightarrow \phi =$ **23.9°**.

Another way is to find the kinetic energy imparted to the electron (the difference in the photon energies), and from this find its speed and momentum; then use this in either (2–3a) or (2–3b).

18. $\frac{h}{\lambda} - \gamma_u m_e u\cos\phi = \frac{h}{\lambda'}\cos\theta$ and $\gamma_u m_e u\sin\phi = \frac{h}{\lambda'}\sin\theta$. Square and add, to eliminate θ

$$\left(\frac{h}{\lambda} - \gamma_u m_e u\cos\phi\right)^2 + (\gamma_u m_e u\sin\phi)^2 = \frac{h^2}{\lambda'^2} \rightarrow \frac{h^2}{\lambda^2} - 2\frac{h}{\lambda}\gamma_u m_e u\cos\phi + (\gamma_u m_e u)^2 = \frac{h^2}{\lambda'^2}$$

But $KE_e = (\gamma_u-1)m_e c^2 = h\frac{c}{\lambda} - h\frac{c}{\lambda'} \Rightarrow \left((\gamma_u-1)m_e c - \frac{h}{\lambda}\right)^2 = \frac{h^2}{\lambda'^2}$.

Set equal: $\left((\gamma_u-1)m_e c - \frac{h}{\lambda}\right)^2 = \frac{h^2}{\lambda^2} - 2\frac{h}{\lambda}\gamma_u m_e u\cos\phi + (\gamma_u m_e u)^2 \rightarrow$

$$((\gamma_u-1)m_e c)^2 - 2\frac{h}{\lambda}(\gamma_u-1)m_e c = -2\frac{h}{\lambda}\gamma_u m_e u\cos\phi + (\gamma_u m_e u)^2$$

or $(\gamma_u-1)^2 m_e c^2 - 2\frac{h}{\lambda}(\gamma_u-1)c = -2\frac{h}{\lambda}\gamma_u u\cos\phi + \gamma_u^2 m_e u^2$.

But $\phi=60°$, $u = 0.45\times10^{8}$m/s $= 0.15c$ and $\gamma_u = \frac{1}{\sqrt{1-(0.15)^2}} = 1.01144$, so this becomes:

$(0.01144)^2(9.11\times10^{-31}\text{kg})c^2 - 2\frac{h}{\lambda}(0.01144)c = -2\frac{h}{\lambda}(1.01144)(0.15c)(0.5)$

$+ (1.01144)^2(9.11\times10^{-31}\text{kg})(0.15c)^2$. Solving: $\frac{h}{\lambda} = 1.62\times10^{-31}c \Rightarrow \lambda =$ **1.37×10^{-11}m**.

19. $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \rightarrow \lambda' - 500\times10^{-9}\text{m} = \frac{6.63\times10^{-34}\text{J}\cdot\text{s}}{(9.11\times10^{-31}\text{kg})(3\times10^{8}\text{m/s})}(1-0)$

$\Rightarrow \lambda' = 500.00243\text{nm}$. $KE_e = \frac{1}{2}mv^2 = h\frac{c}{\lambda} - h\frac{c}{\lambda'} \Rightarrow$

$$v = \sqrt{\frac{2hc}{m}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)} = \sqrt{\frac{2(6.63\times10^{-34}\text{J}\cdot\text{s})(3\times10^{8}\text{m/s})}{9.11\times10^{-31}\text{g}}\left(\frac{1}{5\times10^{-7}\text{m}} - \frac{1}{5.0000243\times10^{-7}\text{m}}\right)} = 2.1\text{km/s}$$

20. $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$. $\Delta\lambda$ is greatest when θ is 180°: $\Delta\lambda = \frac{h}{m_e c}\times 2 = \frac{6.63\times10^{-34}\text{J}\cdot\text{s}}{(9.11\times10^{-31}\text{g})(3\times10^{8}\text{m/s})}2$ $=$ **0.00485nm**. For a proton: $\frac{6.63\times10^{-34}\text{J}\cdot\text{s}}{(1.67\times10^{-27}\text{kg})(3\times10^{8}\text{m/s})}2 =$ **0.00000265nm**. This is very small. Compared to this, even photons in the X–ray range have long wavelengths, and would thus interact with the proton as a wave. The electron collision is more likely to reveal the photon's particle nature.

23. Must eliminate u and λ'. $\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta = \gamma_u m_e u\cos\phi$ and $\frac{h}{\lambda'}\sin\theta = \gamma_u m_e u\sin\phi$.

Divide $\frac{(1/\lambda')\sin\theta}{(1/\lambda) - (1/\lambda')\cos\theta} = \tan\phi$ which becomes $\frac{\sin\theta}{(\lambda'/\lambda) - \cos\theta} = \tan\phi$. But using

$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$, we obtain $\frac{\sin\theta}{\left[\lambda + \frac{h}{m_e c}(1-\cos\theta)\right]/\lambda - \cos\theta} = \tan\phi$

$\rightarrow \frac{\sin\theta}{\left(\frac{h}{mc\lambda}+1\right)(1-\cos\theta)} = \tan\phi \rightarrow \frac{\sin\theta}{1-\cos\theta} = \left(\frac{h}{mc\lambda}+1\right)\tan\phi$. But since

$\sin\theta = 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$ and $1-\cos\theta = 2\sin^2\frac{1}{2}\theta$, this becomes $\cot\frac{1}{2}\theta = \left(\frac{h}{mc\lambda}+1\right)\tan\phi$

25. The initial momentum is zero; a single photon cannot have zero momentum. (b) To conserve momentum, the photons must move in **opposite** directions, with equal momenta h/λ.
The energy of each photon must equal the mass/internal energy of a muon. $h\,c/\lambda = m_\mu c^2$
$\rightarrow \frac{(6.63\times10^{-34}\text{J}\cdot\text{s})(3\times10^{8}\text{m/s})}{\lambda} = (1.88\times10^{-28}\text{kg})(9\times10^{16}\text{m}^2/\text{s}^2) \Rightarrow \lambda =$ **1.18×10^{-14}m**