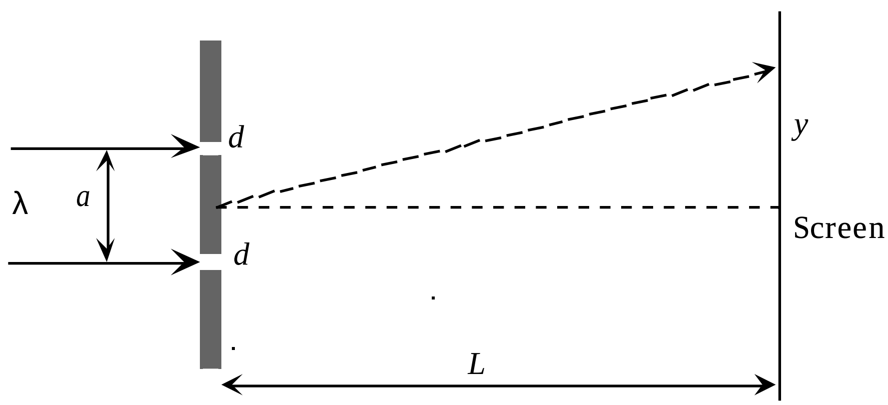
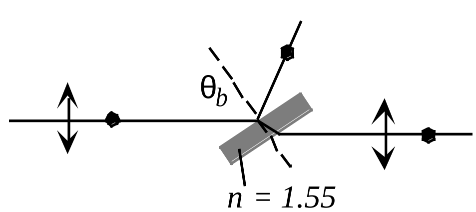
Physics 108 Final Exam (Sample#1)

1. A collimated beam of light with wavelength λ is normally incident on two identical, long slits with width d . The slits are along x-axis in the x-y plane. The centers of the two slits are separated by a distance a in y direction. A screen is placed at a distance L away from the two slits. The surface of the screen is parallel to the x-y plane. Assume that $d^2/\lambda L << 1$.
   (1) (10 points) Find the intensity of the light on the screen as a function of y coordinate;
   (2) (10 points) Show that when $a = d$, the result in (1) is reduced to that for a single slit with width $2d$.



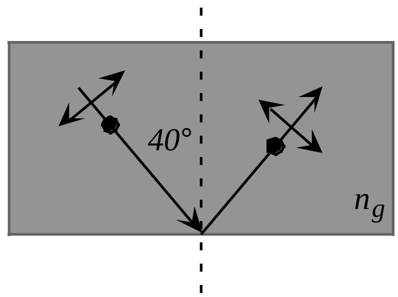
2. A unpolarized light beam is incident from the air onto a glass slide at the Brewster angle θ_b. The index of refraction of the glass slide is $n = 1.55$, and the two surfaces of the slide are parallel to each other.
   (1) (5 points) Find the Brewster angle θ_b;
   (2) (10 points) The transmitted light is subsequently incident on the second surface. For the p-polarized component of the transmitted light, show that the reflectance off the second surface is also zero;
   (3) (15 points) If the intensities for the s-polarized and p-polarized components are the same before passing through the glass slide, how many glass slides does one need to stack together at θ_b in order to make the ratio of the intensity for the p-polarized component to that for the s-polarized component to be 10^5 to 1?



3. An s-polarized light beam with wavelength λ is incident on a surface that separates a glass with refractive index n_g from the air. The incident angle is 40°.

(1) (10 points) Calculate the reflectance $R = |r_s|^2$ for $n_g = 1.4864$ and $n_g = 1.6584$;

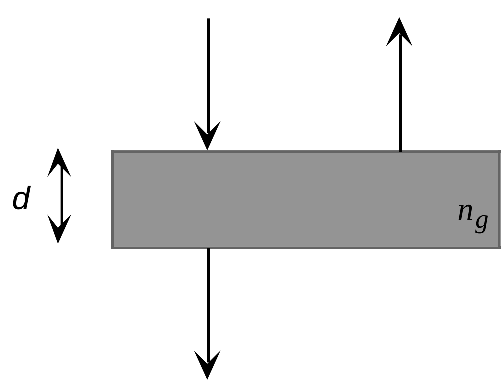
(2) (10 points) For $n_g = 1.6584$, find the amplitude of the transmitted electric field (on the air side) at a distance of 10 λ away from the surface.



4. (20 points) A light beam of wavelength λ is normally incident on a glass slide with refractive index n_g and thickness *d*. The glass slide and the air on both sides form a Fabry-Perot interferometer. Starting from the general result for the transmission coefficient t through a multi-layer film on a substrate, show that the total transmittance $T \equiv |t|^2$ is given by

$$T = \frac{1}{1 + g^2 \sin^2(2\pi n_g d/\lambda)}$$

with $g = 2r/(1 - r^2)$ and $r = (n_g - 1)/(n_g + 1)$.



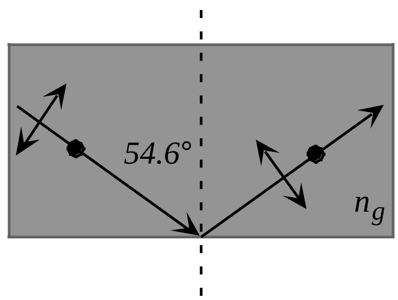
5. A linearly polarized light inside a glass is incident on a glass-air interface at 54.6°. The index of refraction for the glass is $n_g = 1.51$. As a result, the incident beam is *totally* reflected. For the purpose of analyzing the polarization, let the s-polarization along the x-axis, and the p-polarization along the y-axis. The electric field of the linearly polarized incident beam is at 45° from the x-axis.

(1) (5 points) Write down the Jones vector for the incident beam;

(2) (10 points) After the total internal reflection, find the phase shifts for the s-polarized component and the p-polarized component;

(3) (10 points) Write down the Jones vector for the reflected beam;

(4) (10 points) Write down the Jones matrix for such a total reflecting surface.



6. A camera lens is made of four *thin* lenses with $f_1 = +2$ cm, $f_2 = -1$ cm, $f_3 = -5$ cm, and $f_4 = +2.5$ cm. The separation between the first and the second can be neglected; the separation between the third and the second is $d = +1$ cm. The third and the fourth lenses are in contact with each other. A small object is placed at a distance of 10 cm to the left of the camera lens.

(1) (15 points) Find the $\begin{pmatrix} A & B \\ C & C \end{pmatrix}$ matrix for this camera lens set;

(2) (10 points) Find the location of the image with respect to the fourth lens.