Solution to 108 Final Exam (2002)

1-(1):

$$M = M_{f_2} M_d M_{f_1}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

1-(2). F_1: $\quad p = -\frac{D}{C} = \infty$

F_2: $\quad q = -\frac{A}{C} = \infty$.

1-(3):

$$\begin{pmatrix} l_f \\ \alpha_f \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} l_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} l_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3l_0 \\ 0 \end{pmatrix}$$

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2-(1) If the upper slit is blocked,

$$I(y) = I_{inc}\left(\frac{d^2}{\lambda\sqrt{z_0^2+y^2}}\right)\cdot\frac{\sin^2\left(\frac{\pi d}{\lambda}\cdot\frac{y}{\sqrt{z_0^2+y^2}}\right)}{\left(\frac{\pi d}{\lambda}\cdot\frac{y}{\sqrt{z_0^2+y^2}}\right)^2}$$

2-(2) If both slits are open,

$$I(y) = I_{inc}\cdot\frac{d^2}{\lambda\sqrt{z_0^2+y^2}}\cdot\frac{\sin^2\left(\frac{\pi d}{\lambda}\frac{y}{\sqrt{z_0^2+y^2}}\right)}{\left(\frac{\pi d}{\lambda}\frac{y}{\sqrt{z_0^2+y^2}}\right)^2}\cdot$$

$$\cdot\frac{\sin^2\left(\frac{2\pi}{\lambda}\cdot 2d\cdot\frac{y}{\sqrt{z_0^2+y^2}}\right)}{\sin^2\left(\frac{\pi}{\lambda}\cdot 2d\cdot\frac{y}{\sqrt{z_0^2+y^2}}\right)}$$

$$= I_{inc}\cdot\frac{4d^2}{\sqrt{z_0^2+y^2}\cdot\lambda}\cdot\frac{\sin^2\left(\frac{\pi d}{\lambda}\frac{y}{\sqrt{z_0^2+y^2}}\right)}{\left(\frac{\pi d}{\lambda}\frac{y}{\sqrt{z_0^2+y^2}}\right)^2}\cdot\cos^2\left(\frac{2\pi}{\lambda}d\cdot\frac{y}{\sqrt{z_0^2+y^2}}\right)$$

$$= 2\cdot I_{inc}\cdot\frac{d^2}{\lambda\sqrt{z_0^2+y^2}}\cdot\frac{\sin^2\left(\frac{\pi d}{\lambda}\frac{y}{\sqrt{z_0^2+y^2}}\right)}{\left(\frac{\pi d}{\lambda}\cdot\frac{y}{\sqrt{z_0^2+y^2}}\right)^2}\left[1+\cos\left(\frac{4\pi d}{\lambda}\cdot\frac{y}{\sqrt{z_0^2+y^2}}\right)\right]$$

3-(1): Since the thickness increases linearly from left to right, the fringe-to-fringe spacing from left to right is proportional to the thickness change corresponding to two neighboring intensity maxima or two neighboring intensity minima.

Intensity minima for $\lambda_1 = 5000\,\text{Å}$ is given by

$$\frac{4\pi d_1^{(m)}}{\lambda_1} n_0 + \pi = (2m+1)\pi \qquad m = 1, 2, \cdots$$

the thickness change between $d_1^{(m)}$ and $d_1^{(m-1)}$

$$\delta d_1 \equiv d_1^{(m)} - d_1^{(m-1)}$$

is given by

$$\frac{4\pi\,\delta d_1}{\lambda_1} n_0 = 2\pi\,\Delta m = 2\pi$$

$$\therefore\ \delta d_1 = \frac{\lambda_1}{2n_0} \quad ※$$

Similarly,

$$\delta d_2 = \frac{\lambda_2}{2n_0} \quad ※$$

Since $\lambda_1 < \lambda_2$, $\delta d_1 < \delta d_2$, thus the fringes of green color have a smaller fringe-to-fringe spacing.

3-(2) The interference fringes for both colors start as intensity minima on the left side. Since the red color (λ_2) has a larger fringe-to-fringe spacing, its bright fringes progressively shift to the right away from the corresponding bright fringes of green color (λ_1) as the thickness increases. At some point, ~~the~~ a bright fringe at λ_2 will overlap with a dark fringe at λ_1. Let the thickness be $d_1^{(m)}$

$$\frac{4\pi n}{\lambda_1} d_1^{(m)} = 2m\pi \quad \text{- - - -} \quad (1)$$

At this thickness, the green color has its $(m+1)^{th}$ minimum, while the red color has its m^{th} maximum given by

$$\frac{4\pi n}{\lambda_2} d_1^{(m)} + \pi = 2m\pi \quad \text{- - - -} \quad (2)$$

Insert (2) into (1):

$$\frac{4\pi n}{\lambda_1} d_1^{(m)} = \frac{4\pi n}{\lambda_2} d_1^{(m)} + \pi$$

$$\therefore \; d_1^{(m)} = \frac{\lambda_1 \cdot \lambda_2}{4n\,(\lambda_2 - \lambda_1)} = 3750 \text{ Å}$$

4-(1). $\theta_B = \tan^{-1} \dfrac{n_{air}}{n_{glass}} = 33.9^\circ$

4-(2). $\theta_c = \sin^{-1} \dfrac{n_{air}}{n_{glass}} = 42.3^\circ$

4-(3). Since $\tan\theta_B = \dfrac{n_{air}}{n_{glass}} = \sin\theta_c$,

$$\frac{\sin\theta_B}{\sin\theta_c} = \cos\theta_B < 1$$

$$\therefore \; \sin\theta_B < \sin\theta_c$$

$$\Rightarrow \; \theta_c > \theta_B.$$

4-(4) At $\theta_{inc} = \theta_B$, $\cos\theta_2 = \sin\theta_{inc} = \sin\theta_B$,

$$t_p(\theta_{inc} = \theta_B) = \frac{2n_g \cdot \cos\theta_B}{n_{air} \cdot \cos\theta_B + n_g \cdot \cos\theta_2}$$

$$= \frac{2n_g \cos\theta_B}{n_{air} \cdot \cos\theta_B + n_g \cdot \sin\theta_B}$$

$$= \frac{2n_g}{n_{air} + n_g \cdot \tan\theta_B}$$

$$= \frac{2n_g}{n_{air} + n_{air}}$$

$$= \frac{n_g}{n_{air}} > 1$$

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5-(1)

$$\begin{pmatrix} 1+i \\ 1-i \end{pmatrix} = \begin{pmatrix} \sqrt{2}\, e^{i\pi/4} \\ \sqrt{2}\, e^{-i\pi/4} \end{pmatrix} = \sqrt{2}\, e^{i\pi/4} \begin{pmatrix} 1 \\ e^{-i\pi/2} \end{pmatrix}$$

$$= \sqrt{2}\, e^{i\pi/4} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\tilde{E} = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

It is a right-circularly polarized light.

5-(2)

$$M_{\theta+\pi/2} = \begin{bmatrix} \cos^2(\theta+\pi/2) & \sin(\theta+\pi/2)\cos(\pi/2+\theta) \\ \sin(\theta+\pi/2)\cos(\theta+\pi/2) & \sin^2(\theta+\pi/2) \end{bmatrix}$$

$$= \begin{pmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix}$$

5-(3)

$$M_{\theta+\pi/2} M_\theta = \begin{pmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{pmatrix} \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Similarly,

$$M_\theta M_{\theta+\pi/2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$ ※

6-(1)

$$M = \begin{pmatrix} \cos\phi & \frac{\sin\phi}{in} \\ \frac{n\sin\phi}{i} & \cos\phi \end{pmatrix}$$

$$\phi = \frac{2\pi}{\lambda} n d = \frac{2\pi}{5000\text{Å}} \cdot 2 \cdot 100\text{Å} = 14.4^\circ$$

$$M = \begin{pmatrix} 0.97 & -0.124i \\ -0.497i & 0.97 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

$$r = \frac{M_{11} - M_{21} + M_{12} - M_{22}}{M_{11} + M_{21} + M_{12} + M_{22}}$$

$$= \frac{0.97 + i0.497 - i0.124 - 0.97}{0.97 - i0.497 - i0.124 + 0.97}$$

$$= \frac{i0.373}{1.94 - i0.621}$$

$$R = |r|^2 = \frac{0.14}{3.76 + 0.39} = 0.034$$

6-(2) $$t = \frac{2}{M_{11} + M_{21} + M_{12} + M_{22}}$$

$$= \frac{2}{1.94 - i0.621}$$

$$T = |t|^2 = \frac{4}{3.74 + 0.39} = 0.964$$

$$R + T = 1 \quad \#$$