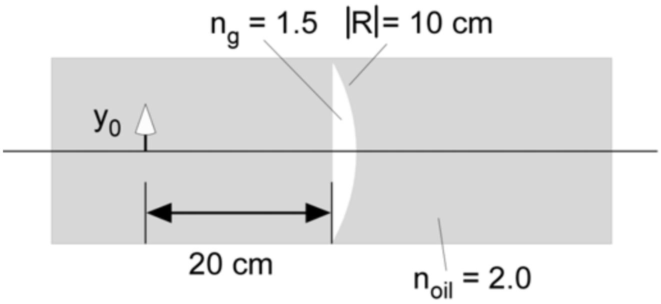
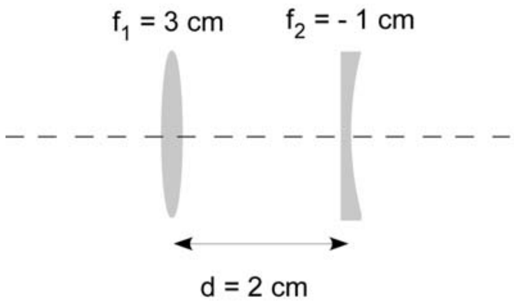
Physics 108 Final Exam (Spring Quarter, 2014)

1. A ***thin*** plano-convex lens with refractive index $n_g = 1.5$ is submerged in oil with refractive index $n_{oil} = 2.0$. A 1-cm high object (y_0) is 20 cm away from the lens.

   1) (**10 points**) Find the focal length of the lens in the oil.
   2) (**10 points**) Find the size of the image.



2. A beam-reducer is made of a ***thin*** converging lens with $f_1 = 3\,\text{cm}$ and a ***thin*** diverging lens with $f_2 = -1\,\text{cm}$, separated by $d = 2\ \text{cm}$.



   1) (**15 points**) Find the ABCD-matrix for such a lens combination.
   2) (**10 points**) Show that both system focal points F_1 an F_2 are at infinity.
   3) (**10 points**) A ray parallel to the optical axis is incident on the first lens from left at a height ℓ_0 as described below

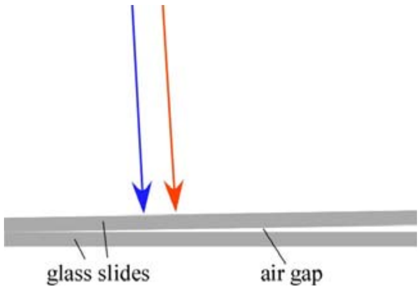
$$\begin{pmatrix} \ell_0 \\ \alpha_0 \end{pmatrix} = \begin{pmatrix} \ell_0 \\ 0 \end{pmatrix}.$$

Show that the outgoing ray after the second lens is also parallel to the optical axis and the height now is reduced by 3 or at $\ell_0/3$,

$$\begin{pmatrix} \ell_f \\ \alpha_f \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \ell_0 \\ 0 \end{pmatrix} = \begin{pmatrix} \ell_0/3 \\ 0 \end{pmatrix}$$

3. Two glass slides are pressed together so that a small wedge-shaped air gap is formed. The refractive index of the air can be taken as $n_{air} = 1$. When the air gap is viewed close to the normal incidence under the illumination of a monochromatic light source with λ_1 = 6000 Å (red), one can see 30 fringes across the *lateral* dimension of the air gap.

1) (**10 points**) Find the thickness change across the *lateral* dimension of the gap.
2) (**5 points**) If the illuminating source is replaced with a monochromatic light with λ_2 = 4500 Å (blue), roughly how many fringes will one see?

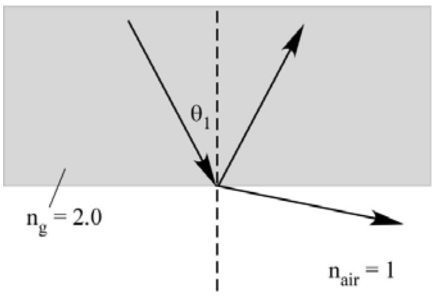


4. An Earth-based CO_2 laser beam with a diameter d = 10 cm and a wavelength λ = 10 μm is pointed at the Moon surface. The distance between the Earth and the Moon is $L = 3.84 \times 10^5$ km.

1) (**10 points**) Find the diameter of the CO_2 laser beam when it reaches the surface of the Moon due to diffraction.
2) (**5 points**) Find the maximum intensity of the CO_2 laser beam at the Moon surface in terms of the initial intensity I_0 of the laser when it leaves the surface of the Earth.

5. A monochromatic light beam, traveling inside a glass with refractive index $n_g = 2$, is incident on a glass-air interface at angle θ_1.

1) (**5 points**) Find the Brewster angle (polarizing angle) θ_{1B} on the glass side.
2) (**10 points**) At the Brewster angle θ_{1B}, find the reflectance (i.e., $R_s = |r_s|^2$) for the TE component (i.e., *s*-polarized).
3) (**10 points**) When $\theta_1 = 45^o$, the beam is totally reflected. For the TM component (i.e., *p*-polarized), find the electric field on the air side in terms of the *p*-polarized component of the incident electric field.



6. (**20 points**) Specify the polarization state for the following four un-normalized Jones vectors,

(a) $\begin{bmatrix} -\mathrm{i} \\ +\mathrm{i} \end{bmatrix}$; (b) $\begin{bmatrix} -1+i \\ 1+i \end{bmatrix}$; (c) $\begin{bmatrix} -1-i \\ 1+i \end{bmatrix}$; (d) $\begin{bmatrix} i \\ 4 \end{bmatrix}$.

7. A linearly polarized light beam with the electric field vector at + 45° from the x-axis passes through a quarter-wave plate (QWP) that is so oriented that the matrix of the quarter-wave plate is given by

$$M_{QWP} = \begin{pmatrix} 1 & 0 \\ 0 & -\mathrm{i} \end{pmatrix}$$

1) (**10 points**) Find the polarization state of the beam after the QWP.
2) (**10 points**) Show that a combination of a quarter-wave plate with OA parallel to x-axis, a linear polarizer with TA at +45° from the x-axis, and another quarter-wave plate with OA parallel to y-axis is a circular polarizer that passes one circularly polarized component of an incident light while blocking the other circularly polarized component.

   (Hint: find the Jones matrix of the combination and show that only one circularly polarized light can pass this device)