Solution for Physics 108 Final (2014)

1-(1) From

$$\frac{n_{oil}}{S_o} + \frac{n_{oil}}{S_i} = (n_g - n_{oil})\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{n_g - n_{oil}}{n_{oil}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

We have the inverse of the focal length f

$$\frac{1}{f} = \frac{n_g - n_{oil}}{n_{oil}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since $R_1 = \infty$, $R_2 = -10\,cm$, $n_g = 1.5$, $n_{oil} = 2.0$

$$\frac{1}{f} = \frac{1.5 - 2.0}{2.0} \cdot (-)\frac{1}{(-10\,cm)} = -\frac{1}{40\,cm}$$

$$\therefore f = -40\,cm$$ ※

1-(2) From $1/s_o + 1/s_i = 1/f$, $\quad s_i = \frac{s_o f}{s_o - f} = -\frac{40}{3}\,cm$

$$y_i = \left(-\frac{s_i}{s_o}\right) \cdot y_o = \frac{2}{3}\,cm$$ ※

2-(1) With refractive index of the air = 1,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{3} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{3} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3} & 2 \\ 0 & 3 \end{pmatrix}$$

2-(2) $p = -D/C = -\infty \Rightarrow$ F_1 at infinity

$q = -A/C = -\infty \Rightarrow$ F_2 at infinity.

2-(3) $$\begin{pmatrix} l_f \\ \alpha_f \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} l_0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} l_0 \\ 0 \end{pmatrix} = \begin{pmatrix} l_0/3 \\ 0 \end{pmatrix}$$ #

3-(1) Near normal incidence, the neighboring fringes correspond to a height (thickness) change δd given by

$$\frac{4\pi}{\lambda_0} n_{air}\, \delta d = 2\pi$$

$$\therefore\ \delta d \Big|_{\text{neighboring fringes}} = \frac{\lambda_0}{2}$$

With $\lambda_1 = 6000\,\text{Å}$, the observation of 30 fringes means that the total thickness change is 30 times δd

$$\Delta d = 30 \times \delta d \Big|_{\text{neighboring fringes @ } \lambda_1}$$

$$= 30 \times \frac{\lambda_1}{2} = 90{,}000\,\text{Å}$$

3-(2) Since $\delta d \Big|_{\text{neighboring fringes @ } \lambda_2 = 4500\,\text{Å}} = \frac{\lambda_2}{2} = 2250$

the number of "blue" fringes will be

$$N = \frac{\Delta d}{\delta d \Big|_{blue}} = \frac{90{,}000\,\text{Å}}{2250\,\text{Å}} = 40$$

4-(1) Due to diffraction effect, the diameter of the CO_2 laser beam on the Moon should be roughly

$$d\,(\text{at Moon}) \cong \left(\frac{2\lambda}{d}\right)\cdot L$$

$$= \frac{2 \times 10^{-5}\ \text{meters}}{10^{-1}\ \text{meters}} \times 3.84 \times 10^{8}\ \text{meters}$$

$$= 7.68 \times 10^{4}\ \text{meters} \ \#$$

4-(2) The intensity of the laser beam at the Moon surface can be determined from energy conservation

$$(\pi/4)\, d^2 \cdot I_0 \cong (\pi/4)\, d^2(\text{at Moon}) \cdot I\,(\text{at Moon})$$

$$\therefore\ I\,(\text{at Moon}) \cong I_0 \cdot \frac{d^2}{d^2(\text{at Moon})}$$

$$= I_0 \cdot \left(\frac{d^2}{2\lambda L}\right)^2 = \left(1.7 \times 10^{-12}\right) I_0$$

5-(1) On the glass side

$$\tan\theta_{1B} = \frac{n_{air}}{n_g} = \frac{1}{2}$$

$$\therefore \theta_{1B} = 26.57^\circ$$

5-(2) From

$$r_s(\theta_{1B}) = \frac{n_g \cos\theta_{1B} - n_{air}\cos\theta_{2B}}{n_g \cos\theta_{1B} + n_{air}\cos\theta_{2B}}$$

$$= \frac{n_g - n_{air}\cdot\tan\theta_{1B}}{n_g + n_{air}\cdot\tan\theta_{1B}} \quad (\cos\theta_{2B} = \sin\theta_{1B})$$

$$= \frac{n_g^2 - n_{air}^2}{n_g^2 + n_{air}^2}$$

$$= \frac{3}{5}$$

$$R_s(\theta_{1B}) = |r_s(\theta_{1B})|^2 = \frac{9}{25} = 0.36$$ #

5-(3) At $\theta_1 = 45°$,

$$\cos\tilde{\theta}_2 = \sqrt{1 - n_g^2 \cdot \sin^2\theta_1 / n_{air}^2}$$

$$= i$$

$$t_p = \frac{E_p^{(air)}}{E_{p,inc}^{(g)}} = \frac{2 n_g \cdot \cos\theta_1}{n_g \cos\tilde{\theta}_2 + n_{air} \cdot \cos\theta_1}$$

$$= \frac{2\sqrt{2}}{2i + \sqrt{2}/2}$$

$$\therefore \left|E_p^{(air)}\right| = \left|E_{p,inc}\right| \cdot \left|\frac{2\sqrt{2}}{\sqrt{2}/2 + i2}\right|$$

$$= \frac{4}{3}\left|E_{p,inc}\right|$$

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6-(a) $\begin{pmatrix} -i \\ +i \end{pmatrix} = (-i) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, linearly polarized with $\alpha = -45°$.

6-(b) $\begin{pmatrix} -1+i \\ 1+i \end{pmatrix} = (-1+i) \begin{pmatrix} 1 \\ \frac{1+i}{-1+i} \end{pmatrix} = (-1+i) \begin{pmatrix} 1 \\ -i \end{pmatrix}$,

Right circularly polarized light.

6-(c) $\begin{pmatrix} -1-i \\ 1+i \end{pmatrix} = (-1-i) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, linearly polarized with $\alpha = -45°$.

6-(d) $\begin{pmatrix} i \\ 4 \end{pmatrix} = i \begin{pmatrix} 1 \\ -4i \end{pmatrix}$, elliptically polarized

\#

7-(c)

$$\tilde{E}_{inc} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos 45^\circ \\ \sin 45^\circ \end{pmatrix}$$

$$M_{Qwp}(OA \parallel x\text{-axis}) = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

$$\tilde{E}_{out} = M_{Qwp}(OA \parallel x\text{-axis})\, \tilde{E}_{inc}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$\therefore$ It is a right-circularly polarized light.

7-(2) Let the Jones matrix for a quarter-wave plate with OA || x-axis be

$$M_{QWP}(OA\|\hat{x}) = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

then the Jones matrix for a quarter-wave plate with OA || y-axis will be

$$M_{QWP}(OA\|\hat{y}) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

The total system Jones matrix is then given by

$$M = M_{QWP}(OA\|\hat{y})\, M_{linear}(+45^\circ)\, M_{QWP}(OA\|\hat{x})$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1/2 \\ i/2 & i/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix}$$

For an incident left-circular polarized light

$$\tilde{E}_{inc} = \tilde{E}_{Left} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

After the polarizing device combination

$$\tilde{E}_{out} = M\,\tilde{E}_{inc}$$

$$= \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \tilde{E}_{inc}.$$

$\therefore$ It passes $\tilde{E}_{Left}$ unscathed.

If $\tilde{E}_{inc} = \tilde{E}_{Right} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

$$\tilde{E}_{out} = M\,\tilde{E}_{Right} = \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{pmatrix} = 0$$

$\therefore$ It ~~[illegible]~~ removes E_{Right} completely.