

23-17. Using Eq. (23-65) for the TE case:

$$R_{TE} = \left| \frac{\cos \theta - \sqrt{(n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_R n_I)}}{\cos \theta + \sqrt{(n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_R n_I)}} \right|^2$$

and for the TM case,

$$R_{TM} = \left| \frac{-[n_R^2 - n_I^2 + i(2n_R n_I)] \cos \theta + (n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_R n_I)}{[n_R^2 - n_I^2 + i(2n_R n_I)] \cos \theta + (n_R^2 - n_I^2 - \sin^2 \theta) + i(2n_R n_I)} \right|^2$$

The given parameters are $n_R = 2.485$ and $n_I = 1.381$. The free computer algebra system Maxima with the TeXmacs front end, can be used to determine the reflectances. Below I show the syntax used and give a table with the output.

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(%i1) (nR:2.484,nI:1.381,k:%pi/180)$
(%i2) for j from 0 step 10 thru 90 do (RTE[j]:float(abs((cos(k*j)-sqrt(nR^2-nI^2-
sin(k*j)*sin(k*j)+%i*2*nR*nI))/(cos(k*j)+sqrt(nR^2-nI^2-
sin(k*j)*sin(k*j)+%i*2*nR*nI)))^2),RTM[j]:float(abs(((nR^2-nI^2+2*i*nR*nI)*cos(k*j)+sqrt(nR^2-nI^2-
sin(k*j)*sin(k*j)+%i*2*nR*nI))/((nR^2-nI^2+2*i*nR*nI)*cos(k*j)+sqrt(nR^2-nI^2-
sin(k*j)*sin(k*j)+%i*2*nR*nI)))^2))
(%o3) done
(%i4) for j from 0 step 10 thru 90 do display(RTE[j],RTM[j])
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θ	0°	30°	50°	70°	90°
R_{TE}	29.3	34.5	45.4	65.7	100
R_{TM}	29.3	24.2	14.9	5.4	100

23-18. Using the Maxima program from the last problem with the input values $n_R = 1.5$ and $n_I = 5.3$ I find,

θ	R_{TE}	R_{TM}
0°	82.5%	82.5%
30°	84.7%	80.1%
60°	90.9%	69.5%

23-19. Given $n_I = 5.3$ at $\lambda = 589.3 \text{ nm}$. (a) $\alpha = 4\pi n_I / \lambda = 4\pi(5.3) / (589.3 \text{ nm}) = 0.113 \text{ nm}^{-1}$

(b) $I = I_0 e^{-\alpha s}$. For $I = 0.01 I_0$, $e^{-\alpha s} = 0.01 \Rightarrow s = (-1/\alpha) \ln(0.01) = 40.75 \text{ nm} = 0.069 \lambda$

23-21. (a) The penetration depth is

$$|z|_{1/e} = \frac{1}{\alpha} = \frac{\lambda}{2\pi} \frac{1}{\sqrt{\sin^2 \theta / n^2 - 1}} = \frac{0.546 \mu\text{m}}{2\pi} \frac{1}{\sqrt{\sin^2(45^\circ) / (1/1.6)^2 - 1}} = 0.164 \mu\text{m}$$

(b) Since irradiance is proportional to the square of the field amplitude and with $\alpha = \frac{1}{|z|_{1/e}} = 6.089 \mu\text{m}^{-1}$,

$$\frac{I}{I_0} = e^{-2\alpha|z|} = e^{-2(6.089 \mu\text{m}^{-1})(1 \mu\text{m})} = 5.1 \times 10^{-6}$$

25-1. (a) Consider,

$$K \equiv n^2 = (n_R + i n_I)^2 = n_R^2 - n_I^2 + 2 i n_I n_R = K_R + i K_I$$

$$K_R = n_R^2 - n_I^2, K_I = 2 n_I n_R$$

Solving these two relations for n_R and n_I proceeds as,

$$K_I = 2 n_I n_R = 2 n_I \sqrt{K_R^2 + n_I^2}$$

$$K_I^2 = 4 n_I^2 (K_R^2 + n_I^2)$$

$$4 n_I^4 + 4 K_R^2 n_I^2 - K_I^2 = 0$$

$$n_I^2 = \frac{-4 K_R \pm \sqrt{16 K_R^2 + 16 K_I^2}}{8} = \frac{-K_R \pm \sqrt{K_R^2 + K_I^2}}{2}$$

To make $n_I^2 > 0$, choose the + sign. Thus,

$$n_I = \left[\frac{-K_R + \sqrt{K_R^2 + K_I^2}}{2} \right]^{1/2}$$

$$n_R^2 = K_R + n_I^2 = \frac{2 K_R}{2} + \frac{-K_R + \sqrt{K_R^2 + K_I^2}}{2} = \frac{K_R + \sqrt{K_R^2 + K_I^2}}{2}$$

$$n_R = \left[\frac{K_R + \sqrt{K_R^2 + K_I^2}}{2} \right]^{1/2}$$

(b) If $K_I \approx K_R$,

$$n_R = \left[\frac{K_I + \sqrt{2 K_I^2}}{2} \right]^{1/2} = \sqrt{K_I} \left(\frac{1 + \sqrt{2}}{2} \right)^{1/2} = 1.099 \sqrt{K_I}$$

$$n_I = \left[\frac{-K_I + \sqrt{2 K_I^2}}{2} \right]^{1/2} = \sqrt{K_I} \left(\frac{-1 + \sqrt{2}}{2} \right)^{1/2} = 0.455 \sqrt{K_I}$$

25-4. Given

$$n = 1 \text{ electron/atom} \quad N = \frac{N_A \rho}{M} = \frac{(6.02 \times 10^{26}) (2.70 \times 10^3)}{26.982} \text{ m}^{-3} = 6.027 \times 10^{28} \text{ m}^{-3} \quad \sigma_0 = 3.54 \times 10^7 / \Omega\text{-m}$$

$$(a) \quad \sigma_0 = \frac{N e^2}{m \gamma} \Rightarrow \gamma = \frac{N e^2}{m \sigma_0} = \frac{(6.027 \times 10^{28}) (1.602 \times 10^{-19})^2}{(9.109 \times 10^{-31}) (3.54 \times 10^7)} \text{ s}^{-1} = 4.80 \times 10^{13} \text{ s}^{-1}$$

$$(b) \quad \omega_p^2 = \frac{N e^2}{m \epsilon_0} = \frac{(6.027 \times 10^{28}) e^2}{m \epsilon_0} = 1.918 \times 10^{32} \text{ s}^{-2} \Rightarrow \omega_p = 1.38 \times 10^{16} \text{ s}^{-1}$$

(c) At $\lambda = 550 \text{ nm}$, $\omega = 2 \pi f = 2 \pi c / \lambda = 3.425 \times 10^{15} \text{ Hz}$. Now,

$$n_R^2 - n_I^2 \equiv K_R = 1 - \frac{\omega_p^2}{\omega^2 + \gamma^2} = -15.347 \quad 2 n_I n_R \equiv K_I = \frac{\gamma}{\omega} \left(\frac{\omega_p^2}{\omega^2 + \gamma^2} \right) = 0.2291$$

Solving these simultaneously as in problem 25-1 gives,

$$n_R = \left[\frac{K_R + \sqrt{K_R^2 + K_I^2}}{2} \right]^{1/2} = 0.0292 \quad n_I = \frac{K_I}{2 n_R} = 3.92$$

$$25-7. (a) \quad \delta_{\text{Al}} = \left(\frac{2}{\sigma \mu_0 \omega} \right)^{1/2} = \left(\frac{2}{3.54 \times 10^7 (4 \pi \times 10^{-7}) 2 \pi \times 6 \times 10^4} \right)^{1/2} \text{ m} = 0.345 \text{ mm}$$

$$(b) \quad \delta_{\text{s.w.}} = \left(\frac{3.54 \times 10^7}{4.3} \right) \times \delta_{\text{Al}} = 0.991 \text{ m} \approx 1 \text{ m}$$

$$\mathbf{25-8.} \quad \delta_{Ag} = \left(\frac{\lambda}{\sigma \mu_0 \pi c} \right)^{1/2} = \left(\frac{0.1}{3 \times 10^7 (4 \pi \times 10^{-7}) \pi (3 \times 10^8)} \right)^{1/2} \text{ m} = 1.68 \times 10^{-6} \text{ m} = 1.68 \mu\text{m}$$

As long as the silver coating is thicker than this the silver-plated brass component would work.

$$\mathbf{25-9.} \quad (\text{a}) \quad I = I_0 e^{-\alpha x} \Rightarrow (I/I_0) = (1/4) = e^{-\alpha x} = e^{-\alpha(3.42\text{m})} \Rightarrow 3.42 \alpha = \ln(4) \Rightarrow \alpha = 0.405 \text{ m}^{-1}$$

$$(\text{b}) \quad (I/I_0) = (1/100) = e^{-(0.405 \text{ m}^{-1})x} \Rightarrow (0.405 \text{ m}^{-1})x = \ln(100) \Rightarrow x = 11.37 \text{ m}$$