Physics 108 Midterm Solutions (2014)

1-(1) For the first refractive surface, $r_1 = +5\,\text{cm}$,
$n_1 = n_{air} = 1$, $n_2 = n_g = 1.5$, $S_o^{(1)} = +30\,\text{cm}$

from $n_1/S_o^{(1)} + n_2/S_i^{(1)} = (n_2 - n_1)/r_1$

$$\frac{1}{30} + \frac{1.5}{S_i^{(1)}} = \frac{0.5}{5} = \frac{1}{10}$$

$$S_i^{(1)} = \frac{1.5}{\left(\frac{1}{10} - \frac{1}{30}\right)} = +22.5\,\text{cm}$$

For the second surface, $r_2 = +10\,\text{cm}$
$n_1 = n_g = 1.5$, $n_2 = n_{air} = 1$, $S_o^{(2)} = -S_i^{(1)} = -22.5\,\text{cm}$

$$\frac{1.5}{-22.5} + \frac{1}{S_i^{(2)}} = \frac{-0.5}{10} = -\frac{1}{20}$$

$$\therefore S_i^{(2)} = +60\,\text{cm}$$

$$M = \left(M\Big|_{2^{nd}\text{ surface}} \right)\left(m\Big|_{1^{st}\text{ surface}} \right)$$

$$= \left(-\frac{\cancel{n_g}}{n_{air}} \cdot \frac{S_i^{(2)}}{S_o^{(2)}} \right)\left(-\frac{\cancel{n_{air}}}{\cancel{n_g}} \cdot \frac{S_i^{(1)}}{S_o^{(1)}} \right) = -2$$

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1-(2) The focal length of such a thin lens

$$\frac{1}{f} = \frac{n_g - n_{air}}{n_{air}} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$= \frac{0.5}{1} \left(\frac{1}{5} - \frac{1}{10} \right)$$

$$= \frac{1}{20}$$

$\therefore$ $f = +20$ cm.

With $s_o = +30$ cm and $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$.

$$s_i = \frac{s_o \cdot f}{s_o - f} = \frac{30 \times 20}{30 - 20} = +60 \text{ cm}$$

2-(1) Method #1: ABCD matrix method

When the two lenses are pressed together, they act like a thin lens with a focal length f_c. Then its ABCD matrix is given by

$$\begin{pmatrix} 1 & 0 \\ -\frac{1}{f_c} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f^{(2)}} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f^{(1)}} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{f^{(2)}} - \frac{1}{f^{(1)}} & 1 \end{pmatrix}$$

$$\therefore \frac{1}{f_c} = \frac{1}{f^{(1)}} + \frac{1}{f^{(2)}}$$

$$\therefore f_c = \frac{f^{(1)} f^{(2)}}{f^{(1)} + f^{(2)}} = \frac{30 \times 60}{30 + 60} = +20\,\text{cm}.$$

Method #2: Using the definition of the focal length f_c.

f_c is the image distance after the second lens for an axial object at $S_0 = +\infty$

before the first lens. The image distance after the first lens is

$$s_i^{(1)}\Big|_{s_o^{(1)}=+\infty} = f^{(1)}$$

For the second lens, $s_o^{(2)} = -s_i^{(1)} = -f^{(1)}$, thus the image distance after the second lens (i.e., f_c) is given by

$$\frac{1}{s_o^{(2)}} + \frac{1}{f_c} = \frac{1}{f^{(2)}}$$

or

$$\frac{1}{-f^{(1)}} + \frac{1}{f_c} = \frac{1}{f^{(2)}}$$

$$\therefore f_c = \frac{f^{(1)} f^{(2)}}{f^{(1)} + f^{(2)}} = \quad +20 \text{ cm}$$

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2-(2) Using ABCD matrix method

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f^{(2)}} & 1 \end{pmatrix} \begin{pmatrix} 1 & 40 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f^{(1)}} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{60} & 1 \end{pmatrix} \begin{pmatrix} 1 & 40 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{30} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 40 \\ -\frac{1}{60} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{30} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{3} & 40 \\ -\frac{1}{36} & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

2-(3) $$s_i = -\frac{A s_o + B}{C s_o + D} = -\frac{-10 + 40}{-\frac{5}{6} + \frac{1}{3}} = +60\,\text{cm}$$

3-(1) From $1/s_o + 1/s_i = -2/R$,

$$s_i = \frac{s_o\,(R/2)}{s_o + (R/2)}\,(-)$$

Since

$$m = \frac{y_i'}{y_i} = -\frac{s_i'}{s_o}$$

we have

$$m = \frac{y_i'}{y_i} = +\frac{R/2}{s_o + R/2}$$

3-(2) With $s_o = 0.5\,m$, $m = +2$, we have

$$2 = \frac{R/2}{s_o + R/2}$$

$$\therefore\ R = -4s_o = -2\,m$$

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4-(1)

$$I_{total}(y,z) = I_1(y,z) + I_2(y,z) + 2\sqrt{I_1(y,z)\, I_2(y,z)}\,\cos(\phi_2 - \phi_1)$$

$$= 2 I_{single\ beam}\left(1 + \cos\left(\frac{2\pi d\, n_{air}}{\lambda_0} \cdot \frac{y}{\sqrt{z^2+y^2}}\right)\right)$$

since the initial phase difference $\phi_2 - \phi_1$ at the slits is zero

4-(2)

$$I_{total} = 2 I_{single\ beam} \times \left(1 + \cos\left[\frac{2\pi d\, n_{air}}{\lambda_0}\frac{y}{\sqrt{z^2+y^2}} + \delta\phi_2\right]\right)$$

~~4-(3) When $\delta\phi_2$ increases from zero the central fringe ... a given fringe with a fixed $\phi_2(y,z) - \phi_1(y,z)$ will move toward towards a larger y value.~~

4-(3) As $\delta\phi_2$ increases from zero, each fringe as characterized by a constant

$$\phi_2(y,z) - \phi_1(y,z)$$

$$= \frac{2\pi d\, n_{air}}{\lambda_0} \cdot \frac{y}{\sqrt{z^2+y^2}} + \delta\phi_2 \quad \cdots \text{①}$$

moves along y-axis to maintain such a phase difference. As a result, if $\delta\phi_2 > 0$, then $\delta y < 0$. This means that the fringes moves downward.

For example, the original central bright fringe corresponds to $\phi_2(y,z) - \phi_1(y,z) = 0$. When $\delta\phi_2 = 0$, this occurs at $y = 0$.

Now when $\delta\phi_2$ becomes positive, this fringe must occur at a negative value of y. So, the fringe moves down.

You can take a difference on Eq. (1).

$$\Delta\left(\phi_2(y,z) - \phi_1(y,z)\right) = 0$$

$$= \frac{2\pi d\, n_{air}}{\lambda_0} \cdot \frac{\Delta y}{z} + \delta\phi_2 \qquad \therefore \Delta y = -\frac{z\lambda_0 \cdot \delta\phi_2}{2\pi \cdot d \cdot n_{air}} < 0$$