3.

(1) With respect to the first surface with
$R_1 = -40\,cm$, $S_o^{(1)} = +40\,cm$,

$$\frac{n_{air}}{S_o^{(1)}} + \frac{n_g}{S_i^{(1)}} = \frac{n_g - n_{air}}{R_1}$$

$$\frac{1}{S_o^{(1)}} + \frac{2}{S_i^{(1)}} = \frac{(2-1)}{-40cm}$$

$$\therefore\ S_i^{(1)} = -40\,cm$$

With respect to the second surface with
$R_2 = -20cm$, $S_o^{(2)} = d - S_i^{(1)} = 60\,cm$

$$\frac{n_g}{S_o^{(2)}} + \frac{n_{air}}{S_i^{(2)}} = \frac{n_{air} - n_g}{R_2}$$

$$\frac{2}{60cm} + \frac{1}{S_i^{(2)}} = \frac{1}{20cm}$$

$$\therefore\ S_i^{(2)} = +60\,cm$$

$$y_i^{(2)} = y_o\left(-\frac{S_i^{(1)}}{S_o^{(1)}}\cdot\frac{n_{air}}{n_g}\right)\left(-\frac{S_i^{(2)}}{S_o^{(2)}}\cdot\frac{n_g}{n_{air}}\right) = y_o\left(-\frac{60}{60}\right)\left(-\frac{(-40)}{40}\right)$$

$$= -y_o$$

(2)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \dfrac{n_g - n_{air}}{n_{air} R_2} & \dfrac{n_g}{n_{air}} \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{n_{air} - n_g}{n_g R_1} & \dfrac{n_{air}}{n_g} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\dfrac{1}{20} & 2 \end{pmatrix} \begin{pmatrix} 1 & 20 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{1}{80} & \dfrac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 20 \\ -\dfrac{1}{20} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \dfrac{1}{80} & \dfrac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{5}{4} & 10 \\ -\dfrac{3}{80} & \dfrac{1}{2} \end{pmatrix}$$

(3)

$$s_i^{(2)} = -\frac{A s_o^{(1)} + B}{C s_o^{(1)} + D} = -\frac{\frac{5}{4} \times 40 + 10}{-\frac{3}{80} \times 40 + \frac{1}{2}} = +60\,\text{cm}.$$

(4)

$$p = -\frac{D}{C} = -\frac{(1/2)}{(-3/80)} = +\frac{40}{3}$$

$$q = -\frac{A}{C} = -\frac{(5/4)}{(-3/80)} = +\frac{100}{3}$$

$$f_1 = -\frac{n_0/n_f}{C} = -\frac{1}{C} = +\frac{80}{3}$$

$$f_2 = -\frac{1}{C} = +\frac{80}{3}$$

$$m = \frac{y_i}{y_0} = -\frac{f_1}{s_0 - p}$$

$$= -\frac{(80/3)}{40 - (40/3)}$$

$$= -\frac{(80/3)}{40\left(1 - \frac{1}{3}\right)}$$

$$= -1$$

18-9. The lens matrix is,

$$\begin{bmatrix} 1 & 0 \\ 1/10 & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1/10 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 5/4 \\ -1/20 & 3/2 \end{bmatrix}$$

$$f_1 = 1/C = -20\,\text{cm},\ f_2 = -1/C = 20\,\text{cm}$$
$$q = -A/C = 10\,\text{cm},\ p = D/C = -30\,\text{cm}$$
$$r = (D-1)/C = -10\,\text{cm}, s = (1-A)/C = -10\,\text{cm}$$

18-12. (a) The lens matrix is,

$$\begin{bmatrix} 1 & 15 \\ 0.5/(-4) & 3/2 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 30 \\ (-0.5)/(4\times 1.5) & 2/3 \end{bmatrix} = \begin{bmatrix} 1/3 & 16/3 \\ -1/6 & 1/3 \end{bmatrix}$$

$$f_1 = 1/C = -6\,\text{in},\ f_2 = -1/C = 6\,\text{in}$$
$$q = -A/C = 2\,\text{in},\ p = D/C = -2\,\text{in}$$
$$r = (D-1)/C = 4\,\text{in},\ s = (1-A)/C = -4\,\text{in}$$

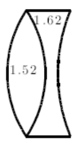
(b) Parallel light focuses at F_2, measured from the output plane (or right surface) by $q = 2\,\text{in}$.

18-14. (a), (b) The lens matrix is

$$\begin{bmatrix} 1 & 0 \\ \frac{0.62}{20} & 1.62 \end{bmatrix}\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{-0.1}{-20\,(1.62)} & \frac{1.52}{1.62} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{-0.52}{(20)(1.52)} & \frac{1}{1.52} \end{bmatrix} = \begin{bmatrix} 0.9764 & 0.96755 \\ 0.009182 & 1.0333 \end{bmatrix}$$

$$f_1 = 1/C = -150\,\text{cm},\ f_2 = -1/C = 150\,\text{cm}$$
$$q = -A/C = 160\,\text{cm},\ p = D/C = -140\,\text{cm}$$
$$r = (D-1)/C = 10\,\text{cm},\ s = (1-A)/C = 10\,\text{cm}$$

(c) Consider each lens separately in air:



$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_1} = \frac{1.52 - 1}{1}\left(\frac{1}{20\,\text{cm}} - \frac{1}{-20\,\text{cm}}\right) = 0.052\,\text{cm}^{-1}$$

$$\frac{1}{f_2} = \frac{1.62 - 1}{1}\left(\frac{1}{-20\,\text{cm}} - \frac{1}{20\,\text{cm}}\right) = -0.062\,\text{cm}^{-1}$$

Equivalent focal length: $\dfrac{1}{f_{\text{eq}}} = \dfrac{1}{f_1} + \dfrac{1}{f_2} = 0.052\,\text{cm}^{-1} - 0.062\,\text{cm}^{-1} \Rightarrow f_{\text{eq}} = -100\,\text{cm}$

4-11. Generally, $\boldsymbol{k}\cdot\boldsymbol{r} = k_x x + k_y y + k_z z$.

(a) For propagation along the z-axis, $k_x = k_y = 0$, So $\boldsymbol{k}\cdot\boldsymbol{r} = k_z z$, with $k_z = 2\pi/\lambda$. The waveform can then be written as,

$$\psi = A\sin(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t) = A\sin(k_z z - \omega t) = A\sin\left[\frac{2\pi}{\lambda}(z - v t)\right] = A\sin 2\pi(z/\lambda - v t)$$

(b) In this case, $k_z = 0$ and $k_x = k_y = |k|/\sqrt{2} = \dfrac{1}{\sqrt{2}}\dfrac{2\pi}{\lambda}$. The general form of the wave is then,

$$\psi = A\sin(\boldsymbol{k}\cdot\boldsymbol{r} \pm \omega t) = A\sin(k_x x + k_y y \pm \omega t) = A\sin\left[\frac{2\pi}{\sqrt{2}\lambda}(x + y \pm v t)\right] = A\sin 2\pi\left(\frac{x}{\sqrt{2}\lambda} + \frac{y}{\sqrt{2}\lambda} \pm v t\right).$$

If one is interested in the wave displacement only on the line $x = y$,

$$\psi = A\sin 2\pi\left(\frac{2x}{\sqrt{2}\lambda} \pm v t\right).$$

(c) In this case $\boldsymbol{k} = \dfrac{k}{\sqrt{3}}(\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}} + \hat{\boldsymbol{z}})$ and $\boldsymbol{k}\cdot\boldsymbol{r} = \dfrac{k}{\sqrt{3}}(x + y + z)$, with $k = 2\pi/\lambda$. The waveform is then,

$$\psi = A\sin(\boldsymbol{k}\cdot\boldsymbol{r} \pm \omega t) = A\sin\left[\frac{k}{\sqrt{3}}(x + y + z) \pm \omega t\right] = A\sin\left[\frac{2\pi}{\sqrt{3}\lambda}(x + y + z \pm v t)\right]$$

4-12. Let $\tilde{z} = a + i\,b$ where a and b are real.

(a) $(\tilde{z} + \tilde{z}\,^*)/2 = (a + i\,b + a - i\,b)/2 = a = \mathrm{Re}(\tilde{z})$

(b) $(\tilde{z} - \tilde{z}\,^*)/2i = (a + i\,b - a + i\,b)/2i = b = \mathrm{Im}(\tilde{z})$

(c) Let $\tilde{z} = e^{i\theta} = \cos\theta + i\sin\theta$ and apply the result from (a): $\cos\theta = \left(e^{i\theta} + e^{-i\theta}\right)/2$

(d) Let $\tilde{z} = e^{i\theta} = \cos\theta + i\sin\theta$ and apply the result from (b): $\sin\theta = \left(e^{i\theta} - e^{-i\theta}\right)/2i$

4-13. (a) Note that $e^{i\pi/2} = \cos(\pi/2) + i\sin(\pi/2) = i$ so that

$$i\,A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \varphi_0)} = e^{i\pi/2}\,A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \varphi_0)} = A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \varphi_0 + \pi/2)}$$

(b) Similarly, $e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1$ so that

$$-A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \varphi_0)} = e^{i\pi}\,A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \varphi_0)} = A\,e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \varphi_0 + \pi)}$$

5-4. One could proceed directly by mimicking the development for the cosine waves leading to Eqs. (5-9) and (5-10). I choose to first convert the given fields to the cosine form and then using those equations. That is,

$$y_1 = 5\sin(\omega t + \pi/2) = 5\cos(\omega t) = 5\cos(0 - \omega t)$$
$$y_2 = 7\sin(\omega t + \pi/3) = 7\cos(\omega t + \pi/3 - \pi/2) = 7\cos(\omega t - \pi/6) = 7\cos(\pi/6 - \omega t)$$

Them using Eqs. (5-9) and (5-10):

$$y_0 = \sqrt{5^2 + 7^2 + 2\cdot 5\cdot 7\cos(\pi/6)} = 11.6$$
$$\tan\alpha = \frac{0 + 7\sin(\pi/6)}{5 + 7\cos(\pi/6)} = \alpha = 0.098\,\pi$$
$$y = 11.6\cos(0.098\,\pi - \omega t) = 11.6\cos(\omega t - 0.098\,\pi) = 11.6\sin(\omega t - 0.098\,\pi + \pi/2)$$
$$y = 11.6\sin(\omega t + 0.402\,\pi)$$