Solution to Physics 108 MT (2015)

1-(1) From $s_o^{(1)} = +40\,cm$, $R_1 = +\infty$, $n_1 = 1$, $n_2 = 2$, and

$$\frac{n_1}{s_o^{(1)}} + \frac{n_2}{s_i^{(1)}} = \frac{n_2 - n_1}{R_1}$$

$\Rightarrow$ $s_i^{(1)} = -n_2 \cdot s_o^{(1)} = -80\,cm$. The image is formed at 80 cm to the left of the flat surface. Thus, $s_o^{(2)} = d - s_i^{(1)} = +100\,cm$; $R_2 = +50\,cm$, $n_1' = n_g = 2$, $n_2' = n_{air} = 1$, then

$$\frac{n_1'}{s_o^{(2)}} + \frac{n_2'}{s_i^{(2)}} = \frac{n_2' - n_1'}{R_2}$$

or

$$\frac{2}{100\,cm} + \frac{1}{s_i^{(2)}} = \frac{1-2}{50\,cm}$$

$\therefore$ $s_i^{(2)} = -25\,cm$, 25 cm to the left of the second surface

$$M = m\Big|_{1^{st}\text{ surface}} \cdot m\Big|_{2^{nd}\text{ surface}}$$

$$= \left(-\frac{n_1}{n_2}\frac{s_i^{(1)}}{s_o^{(1)}}\right)\cdot\left(-\frac{n_1'}{n_2'}\cdot\frac{s_i^{(2)}}{s_o^{(2)}}\right) = \frac{(-80)}{40}\cdot\frac{(-25)}{100}$$

$$= +\frac{1}{2} \; \#$$

1-(2)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{2-1}{50} & 2 \end{pmatrix} \begin{pmatrix} 1 & 20 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{1}{50} & 2 \end{pmatrix} \begin{pmatrix} 1 & 10 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 10 \\ \frac{1}{50} & \frac{6}{5} \end{pmatrix}$$

1-(3)

$$S_i^{(2)} = -\frac{AS_o^{(1)} + B}{CS_o^{(1)} + D}$$

$$= -\frac{40 + 10}{\frac{40}{50} + \frac{60}{50}}$$

$$= -25\ \mathrm{cm}$$ ※

1-(4)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_{d \to 0} = \begin{pmatrix} 1 & 0 \\ \frac{1}{50} & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ \frac{1}{50} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$\therefore$ $f = -50$ cm.

Or

$$f = \left(\frac{n_g - n_{air}}{n_{air}} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \right)^{-1}$$

$$= \left(\frac{2-1}{1} \left(\frac{1}{\infty} - \frac{1}{50} \right) \right)^{-1}$$

$$= \left(-\frac{1}{50} \right)^{-1}$$

$$= -50 \text{ cm}$$

2—(1) Find the image of the van first.

$s_o = 40\text{ m}$, $R = +\infty$, from

$$\frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{R}$$

$s_i = -s_o = -40\text{ m}$, i.e., 40 m behind the flat mirror.

$$y_i = y_o M = (2\text{m})\cdot\left(-\frac{s_i}{s_o}\right) = +2\text{m}$$

the angular size to the driver at 1m in front of the mirror is

$$\alpha = \frac{y_i}{1\text{m} + 40\text{m}} = \frac{2}{41} = 0.049 \text{ radians}$$

2—(2) Now with $R = +80\text{ m}$ and

$$\frac{1}{s_o} + \frac{1}{s_i} = -\frac{2}{R}$$

$\Rightarrow s_i = -20\text{ m}$, i.e. 20 m behind the curved mirror.

$$y_i = y_o \cdot m = (2\text{m})\cdot\left(-\frac{s_i}{s_o}\right) = +1\text{m}$$

$$\alpha' = \frac{y_i}{1\text{m}+20\text{m}} = \frac{1}{21} = 0.048 \text{ radians}$$

2-(3) In this case, the van appears slightly farther away from the viewer than it actually is.

This effect is much more pronounced if R is much smaller than 80m. This is why the right side-mirror on a car or any motor vehicle has a warning sign printed on the front surface.

3-(i) From

$$I_{total} = I_1 + I_2 + 2\sqrt{I_1 \cdot I_2} \cdot \cos(\phi_1 - \phi_2)$$

and

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 \cdot I_2}$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 \cdot I_2},$$

since the slit dimensions are the same, and the illumination is the same, I_1 and I_2 remain unchanged. So do I_{max} and I_{min}.

As to the fringe spacing ~~near~~ near the center of the screen,

$$\delta y = \left(\frac{x}{d}\right)\left(\frac{\lambda_0}{n}\right),$$

when d is reduced by a factor of 10, i.e., $d' = 10/d$,

$$\delta y' = \left(\frac{x}{d'}\right)\left(\frac{\lambda_0}{n}\right) = 10 \cdot \delta y$$

is increased by a factor of 10 #

3-(2) The thickness change in the air gap between two neighboring fringes is

$$\delta d = \frac{\lambda_0}{2\cdot\cos\theta}$$

θ is the incident angle inside the gap, and should be the same as the viewing angle of $60°$. Thus

$$\delta d = \frac{\lambda_0}{2\cdot\cos 60°} = \lambda_0.$$

~~[illegible]~~ Across 10 fringes, we have $9\,\delta d$ in the air gap thickness change

$$\Delta d \equiv 9\,\delta d = 9\times(0.5\,\mu m) = 4.5\,\mu m$$

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3-(3) At normal incidence, those wavelength that yield minimum reflection satisfy

$$\frac{4\pi n}{\lambda_0} d = 2\pi\cdot m$$

$$\lambda_0 = \frac{1}{m}\cdot\frac{4}{3}\ (\mu m)$$

Between $0.35\,\mu m$ and $0.75\,\mu m$, we have $\lambda_0 = \frac{2}{3}\,\mu m,\ \frac{4}{9}\,\mu m$

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