Physics 108 Final Exam (Spring of 2021)

1. **Thin lens:**

   (**15 points**) A *biconvex* (center-thick) thin lens has refractive index $n_g = 1.5$ and radii of curvature $|R_1| = |R_2| = 20\ cm$. It is submerged in a transparent oil with refractive index $n_{oil} = 2.0$. An object $y_0 = 2mm$ in the oil is 20 cm in front of the lens, find the location and linear size of its image y_i.

2. **Diffraction:**

   1) (**5 points**) You have a flashlight that produces a collimated beam with initial diameter $d = 5\ cm$. Let the distance between Davis and Lake Tahoe be 100 miles or 161 km. If you point the flashlight to a mountain in Lake Tahoe, what will be the beam diameter when it reaches the mountain? Assume that the mean wavelength of light is $\lambda_0 = 0.5\ \mu m$.
   2) (**Extra 10 points**) You have a rectangular aperture that measures 100 um in x direction and 20 um in y direction. A collimated light beam with $\lambda_0 = 0.5\ \mu m$, propagating along z-axis, is normally incident on the aperture. A screen is placed 1 m away behind the aperture. Find the dimension of the central bright feature on the screen along the x-axis and y-axis. How does it compare with the original aperture in terms of aspect ratio?

3. **Single-slit diffraction and two-beam interference:**

   In a Young's double-slit experiment, one slit (the top one) has a width d, and the other slit (the bottom one) has a width $2d$. The center-to-center slit separation is $a \gg d$. The slits are in the x-y plane. Let a collimated beam with wavelength λ_0 and intensity I_{inc} normally incident on the slits. A screen is placed at a distance $z = L \gg a$ behind the slits.

   1) (**5 points**) Find the intensity on the screen as a function of the outgoing angle θ when the bottom slit is covered, namely, $I_1(\theta)$, with $\sin\theta = y/\sqrt{y^2 + L^2}$;
   2) (**5 points**) Find the intensity on the screen as a function of the outgoing angle θ when the top slit is covered, namely, $I_2(\theta)$;
   3) (**5 points**) Find the intensity $I(\theta)$ on the screen;
   4) (**Extra 5 points**) Express $I_2(\theta)$ in terms of $I_1(\theta)$.

4. **Reflection and transmission coefficients:**

   In the late afternoon, Sun shines on the sea water off the coast of California at angle $\theta_1 = 60°$. The refractive index of the sea water is $n_{water} = 1.33$.

   1) (**10 points**) Find the reflectance for *s*-polarized and *p*-polarized components of the Sun light. (If you wear a polarizing sun glasses that completely blocks one of the components, you will be able to see fish or other sea lives in the water easily without the glare.)
   2) (**10 points**) When the sea water is calm, you are swimming in the water with a goggle and look up for objects. At what angle *from the plane of the water surface* will you start to see objects above the water? (Hint: this angle is equal to $90° - \theta_2$).
   3) (**10 points**) Show that a light beam inside a transparent material of refractive index n_1 is always 100% reflected when it is incident on the surface of another material of n_2 at angle θ_1 near 90°, regardless the magnitude of n_2 or whether n_2 is complex or not.

5. **Polarization:**

1) (**15 points**) Determine the state of polarization for the following *un*-normalized Jones vector:

(a) $\begin{pmatrix} e^{i\pi/4} \\ e^{-i3\pi/4} \end{pmatrix}$

(b) $\begin{pmatrix} -1+i \\ -i-1 \end{pmatrix}$

(c) $\begin{pmatrix} 4i \\ 2+2i \end{pmatrix}$

2) (***Extra 10 points***) When a left-circularly polarized light $\tilde{E}_L$ and a right-circularly polarized light $\tilde{E}_R$ are superimposed such that the resultant Jones vector, albeit unnormalized, is $\tilde{E}_{out} = \tilde{E}_L + e^{i\delta}\tilde{E}_R$. Show that $\tilde{E}_{out}$ is a linearly polarized light along the direction that is $\delta/2$ from the x-axis.

6. **Polarizing devices:**

1) (**5 points**) A linearly polarized light with the electric field oriented at $-120°$ from the x-axis. Find its normalized Jones vector.

2) (**5 points**) A linearly polarized light at $-45°$ from the x-axis passes through a quarter-wave plate with its fast axis (FA) parallel to the x-axis, what is the state of polarization after the wave-plate?

3) (**10 points**) A linearly polarized light along the y-axis passes through a half-wave plate with its fast axis (FA) at $+45°$ from the x-axis, what is the state of polarization after the wave-plate?