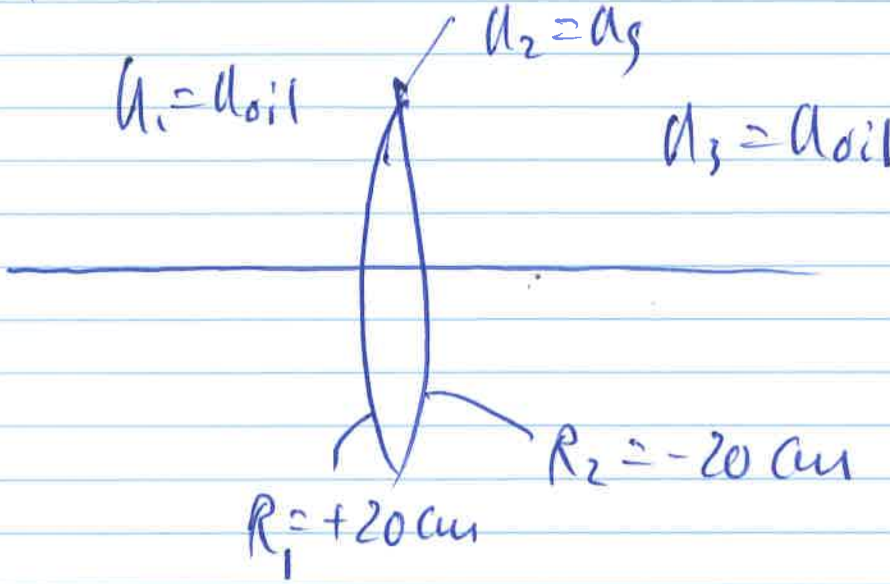
Solutions for Physics 108 Final Exam. (Spring, 2021)

1. Thin lens:



Using the general thin lens equation,

$$\frac{n_1}{s_o} + \frac{n_3}{s_i} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \quad \cdots (1)$$

Since $n_3 = n_1 = n_{oil}$, this is reduced to

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \cdots (2)$$

$$f = \frac{n_{oil}}{(n_g - n_{oil})(1/R_1 - 1/R_2)} = -40 \text{ cm}$$

It is a negative lens. From Eq. (2),

$$s_i = \frac{s_o \cdot f}{s_o - f} = (-) \cdot \frac{20\text{cm} * 40\text{cm}}{20\text{cm} - (-40\text{cm})} = -\frac{40}{3} \text{ cm}.$$

The linear size of the virtual image y_i is

$$y_i = y_o \cdot m = y_o\left(-\frac{s_i}{s_o}\right)$$

since $a_3 = a_1$

$$= 2\,\text{mm} \times (-)\,\frac{(-40\,\text{cm}/3)}{20\,\text{cm}}$$

$$= +\,4/3\ \text{mm}.$$

Or

$$y_i = y_o \cdot m = y_o\left(-\frac{f}{s_o - f}\right)$$

$$= 2\,\text{mm} \cdot (-) \cdot \frac{-40\,\text{cm}}{+60\,\text{cm}}$$

$$= +\,4/3\ \text{mm}$$

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2. Diffraction:

2-1. The far-field diverging angle is

$$\Delta\theta = 2\times\left(\lambda_0/d\right) = \frac{2\lambda_0}{d}$$

At a distance L away, the beam diameter is given by

$$D = \Delta\theta \cdot L$$

With $\lambda_0 = 0.5\ \mu m = 5\times10^{-7}\ m$, $d = 5\ cm = 5\times10^{-2}\ m$, $L = 1.61\times10^{5}\ m$,

$$D = \frac{2\times5\times10^{-7}\ m\times1.61\times10^{5}\ m}{5\times10^{-2}\ m}$$

$$= 3.22\ m,$$

much larger than the original flashlight beam diameter.

2-2. We consider Fraunhofer diffraction along each direction separately.

Along x-direction, $d_x = 100\,\mu m$. At $L = 1m$ away, the width of the central bright feature along x direction on the screen is

$$D_x = 2 * L * \left(\frac{\lambda_0}{d_x}\right) = \frac{2 \times 1m \times 0.5\,\mu m}{100\,\mu m}$$

$$= 1cm.$$

Along y-direction, $d_y = 20\,\mu m$. At $L = 1m$, the width of the central bright feature along y direction on the screen is

$$D_y = 2 * L * \left(\frac{\lambda_0}{d_y}\right) = \frac{2 \times 1m \times 0.5\,\mu m}{20\,\mu m}$$

$$= 5\,cm$$

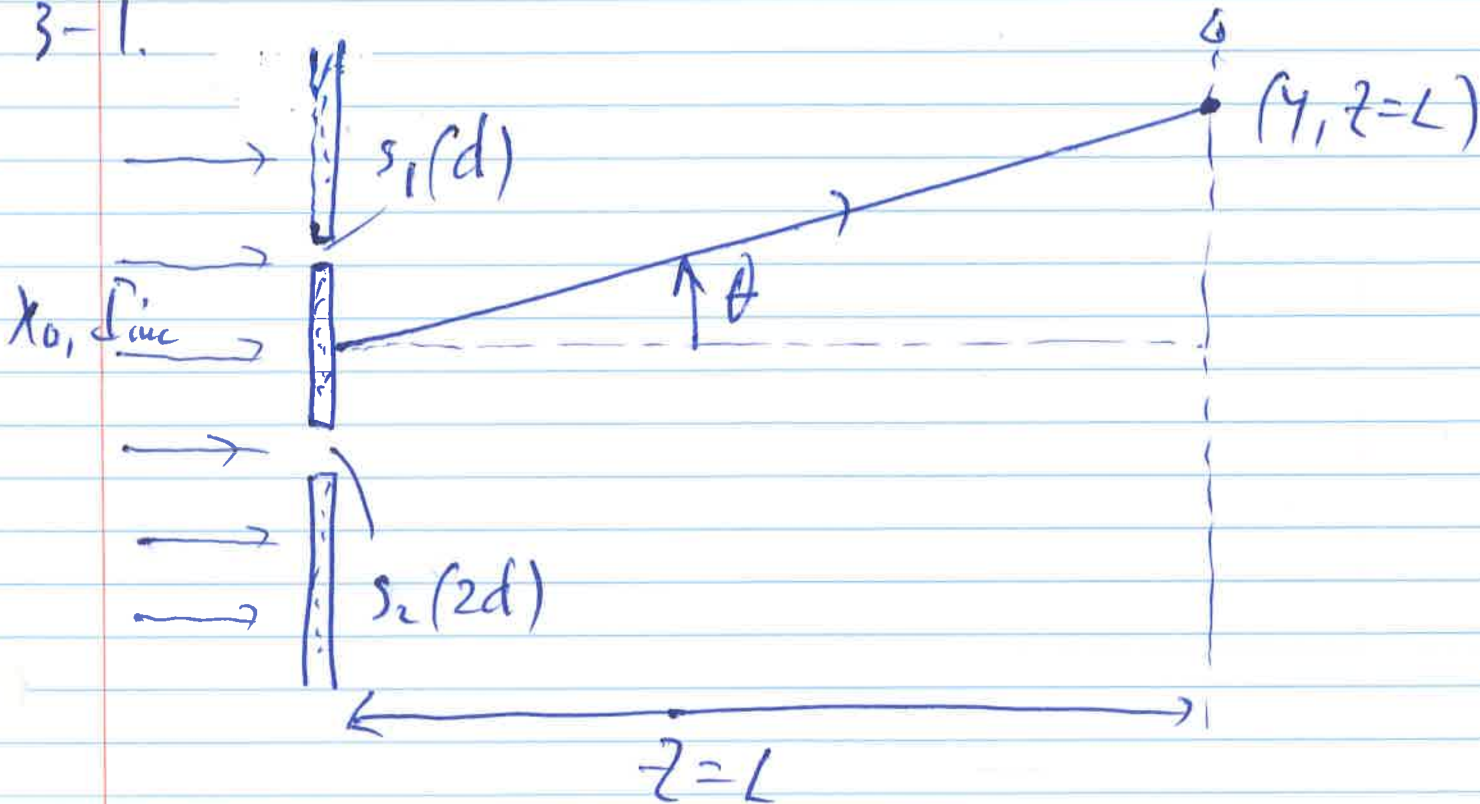
Thus,

$$\frac{D_x}{D_y} = \frac{d_y}{d_x}$$

i.e., the aspect ratio is reversed.

3. Single-slit diffraction and two-beam interference.

3-1.



$$I_1(\theta) = I_{inc}\left(\frac{d^2}{\lambda_0 L}\right)\cdot\frac{\sin^2\left(\frac{\pi d}{\lambda_0}\sin(\theta)\right)}{\left(\frac{\pi d}{\lambda_0}\sin\theta\right)^2}\Bigg|_{d_1=d}$$

3-2.

$$I_2(\theta) = I_{inc}\left(\frac{(2d)^2}{\lambda_0 L}\right)\cdot\frac{\sin^2\left(\frac{\pi\cdot 2d}{\lambda_0}\sin\theta\right)}{\left(\frac{\pi 2d}{\lambda_0}\sin\theta\right)^2}\Bigg|_{d_2=2d}$$

3-3.

$$I(\theta) = I_1(\theta) + I_2(\theta) + 2\sqrt{I_1(\theta)\, I_2(\theta)} \cdot \cos\left(\frac{2\pi a}{\lambda_0}\sin\theta\right)$$

3-4.

$$I_2(\theta) = I_{inc}\left(\frac{4d^2}{\lambda_0 L}\right) \cdot \frac{\left(4\sin^2\left(\frac{\pi d}{\lambda_0}\sin\theta\right)\cos^2\left(\frac{\pi d}{\lambda_0}\sin\theta\right)\right)}{4\left(\frac{\pi d}{\lambda_0}\sin\theta\right)^2}$$

$$= 4\cos^2\left(\frac{\pi d}{\lambda_0}\sin\theta\right) \cdot I_1(\theta)$$

Thus

$$I(\theta) = I_1(\theta)\left[1 + 4\cos^2\left(\frac{\pi d}{\lambda_0}\sin\theta\right) + 4\left|\cos\left(\frac{\pi d}{\lambda_0}\sin\theta\right)\right| \times \cos\left(\frac{2\pi a}{\lambda_0}\sin\theta\right)\right]$$

when $\theta \sim 0$,

$$I(\theta) = I_1(\theta) \cdot \left[5 + 4\cos\left(\frac{2\pi a}{\lambda_0}\sin\theta\right)\right]$$

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4. Reflection and transmission coefficients

4-1. $n_1 = n_{air} = 1$, $n_2 = n_{water} = 1.33$, $\theta_1 = 60°$. From Snell's refraction law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$,

$$\theta_2 = \sin^{-1}(n_1 \sin\theta_1 / n_2) = 40.6°$$

$$\cos\theta_1 = 0.50; \quad \cos\theta_2 = 0.76$$

$$R_s(\theta_1 = 60°) = |r_s(\theta_1) = 60°|^2$$

$$= \left| \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2} \right|^2$$

$$= \left| \frac{0.5 - 1.33 \times 0.76}{0.5 + 1.33 \times 0.76} \right|^2 = 0.11$$

$$R_p(\theta_1 = 60°) = |r_p(\theta_1 = 60°)|^2$$

$$= \left| \frac{n_1\cos\theta_2 - n_2\cos\theta_1}{n_1\cos\theta_2 + n_2\cos\theta_1} \right|^2$$

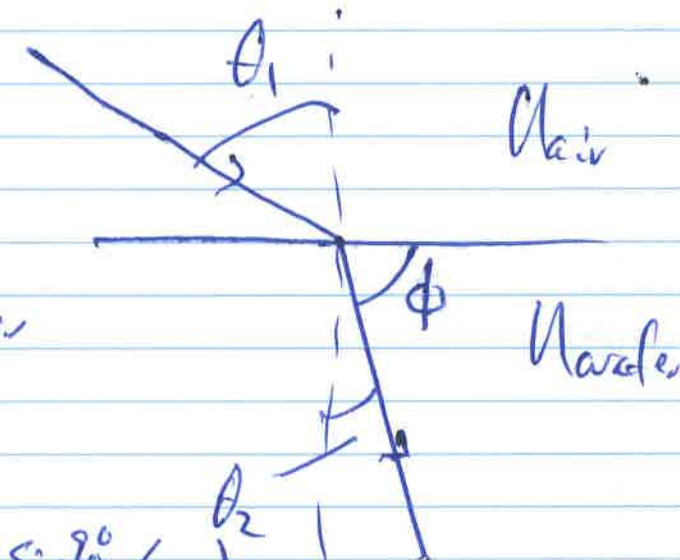
$$= \left| \frac{0.76 - 1.33 \times 0.5}{0.76 + 1.33 \times 0.5} \right|^2 = 4.4 \times 10^{-3}$$

$$= 0.0044$$

4-2 The angle in the sea water from the plane of the water surface, ϕ, is equal to $90 - \theta_2$ where θ_2 is the refraction angle of the light from an object above the water surface having an incidence angle θ_1:

The largest θ_2 is obtained with $\theta_1 = 90^\circ$.

Thus the smallest ϕ from the plane of the water surface is



$$\phi_{min} = 90^\circ - \theta_2(\text{Max})$$
$$= 90^\circ - \sin^{-1}\left(n_{air}\sin 90^\circ / n_{water}\right)$$
$$= 90^\circ - \sin^{-1}\left(\frac{1}{n_{water}}\right)$$
$$= 41.2^\circ$$

So you need to lift the eye from the horizontal plane over 41.2° in order to start to see objects above the sea water.

4-3. For s-polarized light,

$$r_{s,12}(\theta_1=90^\circ) = \frac{n_1\cos 90^\circ - n_2\cos\theta_2}{n_1\cos 90^\circ + n_2\cos\theta_2}$$

$$= -1$$

$$\therefore R_s(\theta_1=90^\circ) = |r_{s,12}(\theta_1=90^\circ)|^2 = 1$$

For p-polarized light,

$$r_{p,12}(\theta_1=90^\circ) = \frac{n_1\cos\theta_2 - n_2\cos 90^\circ}{n_1\cos\theta_2 + n_2\cos 90^\circ}$$

$$= +1$$

$$\therefore R_p(\theta_1=90^\circ) = |r_{p,12}(\theta_1=90^\circ)|^2$$

$$= 1$$

Even when n_2 is complex, or n_2 is less than n_1 such that $\cos\theta_2$ is complex, $r_{s,12}(\theta_1=90^\circ) = -1$, and $r_{p,12}(\theta_1=90^\circ) = +1$, thus

$$R_s(\theta_1=90^\circ) = R_p(\theta_1=90^\circ) = 1$$

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5. Polarization:

5-1. (a)

$$\begin{pmatrix} e^{i\pi/4} \\ e^{-i3\pi/4} \end{pmatrix} = e^{i\pi/4} \begin{pmatrix} 1 \\ e^{-i\pi} \end{pmatrix} = \sqrt{2}\, e^{i\pi/4} \cdot \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

It is a linearly polarized along an angle from the x-axis

$$\alpha = \tan^{-1}\left(\frac{-1/\sqrt{2}}{+1/\sqrt{2}}\right) = -45^\circ$$

(b) $$\begin{pmatrix} -1+i \\ -i-1 \end{pmatrix} = \begin{pmatrix} \sqrt{2}\, e^{i3\pi/4} \\ \sqrt{2}\, e^{i5\pi/4} \end{pmatrix} = 2e^{i3\pi/4} \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$$

It is a left-circularly polarized light.

(c) $$\begin{pmatrix} 4i \\ 2+2i \end{pmatrix} = \begin{pmatrix} 4e^{i\pi/2} \\ 2\sqrt{2}\, e^{i\pi/4} \end{pmatrix} = \sqrt{24}\, e^{i\pi/2} \begin{pmatrix} \sqrt{2/3} \\ \sqrt{1/3}\, e^{-i\pi/4} \end{pmatrix}$$

It is an elliptically polarized light.

5-2.

$$\tilde{E}_{out} = \tilde{E}_L + e^{i\delta}\tilde{E}_R$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} + e^{i\delta}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$$

$$= e^{i\delta/2}\frac{1}{\sqrt{2}}\begin{pmatrix}e^{i\delta/2} + e^{-i\delta/2}\\ (-i)\left(e^{i\delta/2} - e^{-i\delta/2}\right)\end{pmatrix}$$

$$= e^{i\delta/2}\cdot\sqrt{2}\cdot\begin{pmatrix}\left(e^{i\delta/2} + e^{-i\delta/2}\right)/2\\ \left(e^{i\delta/2} - e^{-i\delta/2}\right)/2i\end{pmatrix}$$

$$= \left(\sqrt{2}e^{i\delta/2}\right)\begin{pmatrix}\cos\delta/2\\ \sin\delta/2\end{pmatrix}$$

It is a linearly polarized light, making an angle α from the x-axis:

$$\alpha = \delta/2$$

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6. Polarizing devices

6-1. $$\tilde{E} = \begin{pmatrix} \cos(-120^\circ) \\ \sin(-120^\circ) \end{pmatrix} = \begin{pmatrix} -1/2 \\ -\sqrt{3}/2 \end{pmatrix}$$

6-2. $$\tilde{E}_{inc} = \begin{pmatrix} \cos(-45^\circ) \\ \sin(-45^\circ) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$M_{QWP}(FA \parallel x\text{-axis}) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

$$\tilde{E}_{out} = M_{QWP}(FA \parallel x\text{-axis})\, \tilde{E}_{inc}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

it is a right-circularly polarized light.

6-3. $\tilde{E}_{inc} = \begin{pmatrix} \cos 90^\circ \\ \sin 90^\circ \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

M_{HWP} (FA @ $+45^\circ$ from x-axis)

$$= \begin{pmatrix} \cos(2*45^\circ) & \sin(2*45^\circ) \\ \sin(2*45^\circ) & -\cos(2*45^\circ) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Thus

$$\tilde{E}_{out} = M_{HWP} \tilde{E}_{inc}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

It is a linearly polarized light along x-axis