

4-11. Generally, $\mathbf{k} \cdot \mathbf{r} = k_x x + k_y y + k_z z$.

(a) For propagation along the z -axis, $k_x = k_y = 0$. So $\mathbf{k} \cdot \mathbf{r} = k_z z$, with $k_z = 2\pi/\lambda$. The waveform can then be written as,

$$\psi = A \sin(\mathbf{k} \cdot \mathbf{r} - \omega t) = A \sin(k_z z - \omega t) = A \sin\left[\frac{2\pi}{\lambda}(z - vt)\right] = A \sin 2\pi(z/\lambda - vt)$$

(b) In this case, $k_z = 0$ and $k_x = k_y = |k|/\sqrt{2} = \frac{1}{\sqrt{2}} \frac{2\pi}{\lambda}$. The general form of the wave is then,

$$\psi = A \sin(\mathbf{k} \cdot \mathbf{r} \pm \omega t) = A \sin(k_x x + k_y y \pm \omega t) = A \sin\left[\frac{2\pi}{\sqrt{2}\lambda}(x + y \pm vt)\right] = A \sin 2\pi\left(\frac{x}{\sqrt{2}\lambda} + \frac{y}{\sqrt{2}\lambda} \pm vt\right).$$

If one is interested in the wave displacement only on the line $x = y$,

$$\psi = A \sin 2\pi\left(\frac{2x}{\sqrt{2}\lambda} \pm vt\right).$$

(c) In this case $\mathbf{k} = \frac{k}{\sqrt{3}}(\hat{x} + \hat{y} + \hat{z})$ and $\mathbf{k} \cdot \mathbf{r} = \frac{k}{\sqrt{3}}(x + y + z)$, with $k = 2\pi/\lambda$. The waveform is then,

$$\psi = A \sin(\mathbf{k} \cdot \mathbf{r} \pm \omega t) = A \sin\left[\frac{k}{\sqrt{3}}(x + y + z) \pm \omega t\right] = A \sin\left[\frac{2\pi}{\sqrt{3}\lambda}(x + y + z \pm vt)\right]$$

4-12. Let $\tilde{z} = a + ib$ where a and b are real.

(a) $(\tilde{z} + \tilde{z}^*)/2 = (a + ib + a - ib)/2 = a = \text{Re}(\tilde{z})$

(b) $(\tilde{z} - \tilde{z}^*)/2i = (a + ib - a + ib)/2i = b = \text{Im}(\tilde{z})$

(c) Let $\tilde{z} = e^{i\theta} = \cos\theta + i\sin\theta$ and apply the result from (a): $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$

(d) Let $\tilde{z} = e^{i\theta} = \cos\theta + i\sin\theta$ and apply the result from (b): $\sin\theta = (e^{i\theta} - e^{-i\theta})/2i$

4-13. (a) Note that $e^{i\pi/2} = \cos(\pi/2) + i\sin(\pi/2) = i$ so that

$$i A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = e^{i\pi/2} A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0 + \pi/2)}$$

(b) Similarly, $e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1$ so that

$$-A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = e^{i\pi} A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0)} = A e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t + \varphi_0 + \pi)}$$

5-4. One could proceed directly by mimicking the development for the cosine waves leading to Eqs. (5-9) and (5-10). I choose to first convert the given fields to the cosine form and then using those equations. That is,

$$y_1 = 5 \sin(\omega t + \pi/2) = 5 \cos(\omega t) = 5 \cos(0 - \omega t)$$

$$y_2 = 7 \sin(\omega t + \pi/3) = 7 \cos(\omega t + \pi/3 - \pi/2) = 7 \cos(\omega t - \pi/6) = 7 \cos(\pi/6 - \omega t)$$

Then using Eqs. (5-9) and (5-10):

$$y_0 = \sqrt{5^2 + 7^2 + 2 \cdot 5 \cdot 7 \cos(\pi/6)} = 11.6$$

$$\tan \alpha = \frac{0 + 7 \sin(\pi/6)}{5 + 7 \cos(\pi/6)} = \alpha = 0.098 \pi$$

$$y = 11.6 \cos(0.098 \pi - \omega t) = 11.6 \cos(\omega t - 0.098 \pi) = 11.6 \sin(\omega t - 0.098 \pi + \pi/2)$$

$$y = 11.6 \sin(\omega t + 0.402 \pi)$$