1-(a) For a uniformly distributed spherical volume of charge Q and radius R,

$$\vec{E}(\vec{r};\ |\vec{r}| \leq R) = \frac{1}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \cdot Q \left(\frac{r}{R}\right)^3$$

$$\vec{E}(\vec{r};\ |\vec{r}| \geq R) = \frac{1}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} Q$$

Specialized to point O:

$$\vec{E}_0 = \vec{E}_0^{(1)} + \vec{E}_0^{(2)} = \vec{E}_0^{(2)} = \frac{1}{4\pi\varepsilon_0} \frac{(-\hat{i})}{(2R)^2} Q$$

1-(b) Specialized to point A:

$$\vec{E}_A = \vec{E}_A^{(1)} + \vec{E}_A^{(2)} = \frac{1}{4\pi\varepsilon_0} \frac{\hat{i}}{(R/2)^2} Q \left(\frac{R/2}{R}\right)^3$$

$$+ \frac{1}{4\pi\varepsilon_0} \frac{(-\hat{i})}{(3R/2)^2} \cdot Q$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\hat{i}}{R^2} \frac{Q}{18}$$

1-(c) Specialized to point B:

$$\vec{E}_B = \vec{E}_B^{(1)} + \vec{E}_B^{(2)} = \frac{1}{4\pi\varepsilon_0} \frac{\hat{i}}{(3R)^2} Q + \frac{1}{4\pi\varepsilon_0} \frac{\hat{i}}{R^2} Q$$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{\hat{i}}{R^2}\left(\frac{10}{9}Q\right)$$

1-(d) Specialized to point C:

$$\vec{E}_C = \vec{E}_C^{(1)} + \vec{E}_C^{(2)} = \frac{1}{4\pi\varepsilon_0}\frac{\hat{j}}{(2R)^2}Q$$

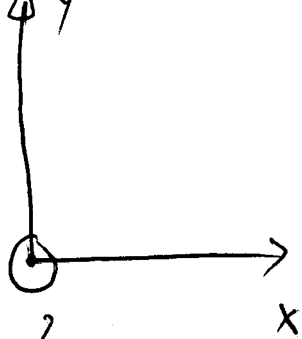
$$+ \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(2R)^2+(2R)^2}\cdot\frac{-\hat{i}+\hat{j}}{\sqrt{2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\left[\left(-\frac{1}{8\sqrt{2}}\right)\hat{i} + \left(\frac{1}{4}+\frac{1}{8\sqrt{2}}\right)\hat{j}\right]$$

2-(a) Immediately after the switch is closed, the current flows clockwise with a magnitude

$$I = \frac{\mathcal{E}}{R} = \frac{12V}{5\Omega} = 2.4A$$

With the current $I = 2.4A$ flowing downward from a to b, the metal bar experiences a magnetic force



$$\vec{F}^{(M)} = I\,\vec{ab} \times \vec{B}$$

$$= (2.4A)(0.8m)(1.5T) \cdot (-\hat{j}) \times (-\hat{k})$$

$$= 2.88N\,\hat{i}$$

Thus the acceleration is to the right, or, $+\hat{i}$, with a magnitude

$$\vec{a} = \frac{\vec{F}^{(M)}}{m} = \frac{2.88N\,\hat{i}}{0.9kg} = 3.2\,m/s^2\,\hat{i}$$

2-(b) When the metal bar reaches a speed of 2m/s to the right, we have

$$\vec{v} = (2\,m/s)\,\hat{i}$$

In the magnetic field, a motion emf $\mathcal{E}_{ind}$ is induced along the metal bar from b to a with a magnitude

$$\mathcal{E}_{ind} = BLv = (1.5T)(0.8m)(2m/s)$$

$$= 2.4 \text{ volts.}$$

As a result, the clockwise current now is reduced to

$$I' = \frac{\mathcal{E} - \mathcal{E}_{induced}}{R} = \frac{12V - 2.4V}{5\Omega}$$

$$= 1.92A. \Rightarrow \vec{a}' = \hat{i}\,\frac{I'lB}{m} = 2.56\,m/s^2\,\hat{i}$$

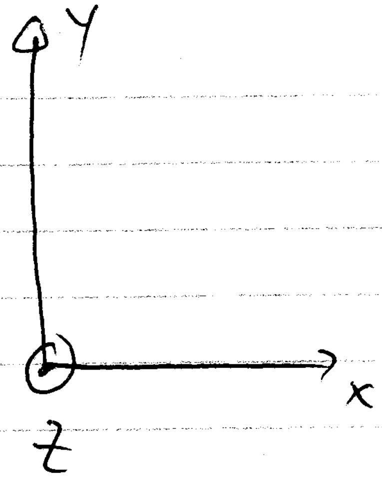
2-(c) As the metal bar gains in speed, the motion emf $\mathcal{E}_{ind} = BLv$ also increases, but produces the opposite effect as the original battery (12V). When the motion emf $\mathcal{E}_{ind} = BLv_t$ reaches the value of the battery, the clockwise current becomes zero, thus $\vec{F}^{(M)} = I\,\vec{ab} \times \vec{B}$ becomes zero also, and the acceleration

$\vec{a} = \vec{F}^{(M)}/m$ becomes zero. From this point on, the metal bar moves at the constant velocity $\vec{v}_t = v_t \hat{i}$ that makes $\mathcal{E}_{ind} = BLv_t$ equal to $\mathcal{E} = 12V$. v_t is then the value of the terminal velocity $\vec{v}_t = v_t \hat{i}$, and is given by

$$\mathcal{E}_{ind} = BL\,v_t = \mathcal{E} = 12 \text{ volts}$$

$$\therefore \; v_t = \frac{12 \text{ volts}}{(1.5T)(0.8\,m)} = 10\ m/s.$$

3-(a) Every small line segment on the current-carrying circular wire produces a magnetic field at A that points into the paper (or along $-\hat{k}$).



$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I \cdot dl}{R^2} \left(-\hat{k}\right)$$

So the total magnetic field at A is

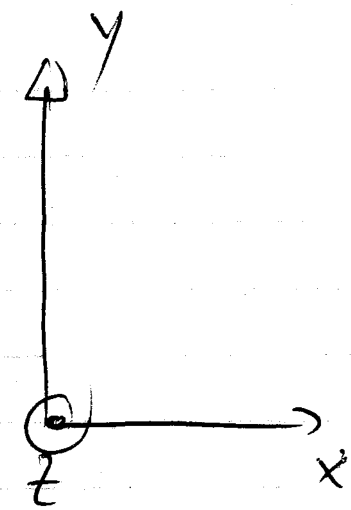
$$\vec{B}_A = \oint d\vec{B} = \frac{\mu_0 I}{2R} \left(-\hat{k}\right)$$

You can obtain this result from the general result of the magnetic field on the axis of a circular-current-carrying wire loop with radius R

$$\vec{B}(z) = \frac{\mu_0 I \cdot R^2}{2\left(z^2+R^2\right)^{3/2}} \left(-\hat{k}\right)$$

$$\vec{B}_A = \vec{B}(z=0) = \frac{\mu_0 I}{2R} \left(-\hat{k}\right)$$

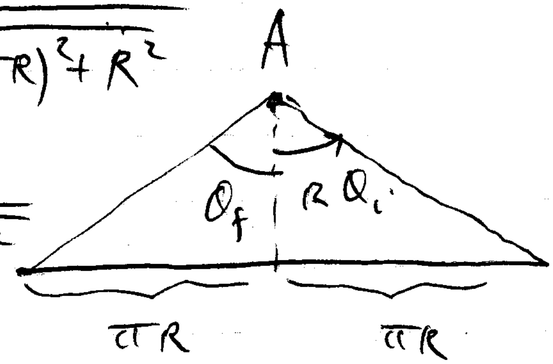
3-(b) Again, each line segment along the straightened wire ($2\pi R$ in total length) produces a magnetic field at point A that points into the paper. The total magnitude



$$\vec{B}_A = (-\hat{k}) \frac{\mu_0 I}{4\pi R} \left(\sin\theta_f - \sin\theta_i \right)$$

$$= (-\hat{k}) \frac{\mu_0 I}{4\pi R} \cdot 2 \cdot \frac{\pi R}{\sqrt{(\pi R)^2 + R^2}}$$

$$= (-\hat{k}) \frac{\mu_0 I}{2R} \cdot \frac{1}{\sqrt{1+\pi^2}}$$



3-(c) Clearly the magnetic field produced by a circular wire of radius R is larger by a factor $(1+\pi^2)^{1/2}$ than that produced by a straight wire of the same length yet carrying the same current. This is because that the magnetic field from a small line segment is proportional to the inverse of its distance squared to the field point. Every segment on the circular wire is at R away from A; but most segments on a straight wire are farther away from the field point A than R.

4. After S is closed, the metal bar acts like a 10Ω resistor in parallel with the other 10Ω resistor in the circuit. As a result, a current will flow downward through the metal bar. It should be one half of the current through the 25Ω resistor.

$$I\Big|_{\text{through } 25\Omega} = \frac{\mathcal{E}}{25 + \left(\frac{1}{10} + \frac{1}{10}\right)^{-1}} = \frac{\mathcal{E}}{30}$$

Thus the current through the metal bar is

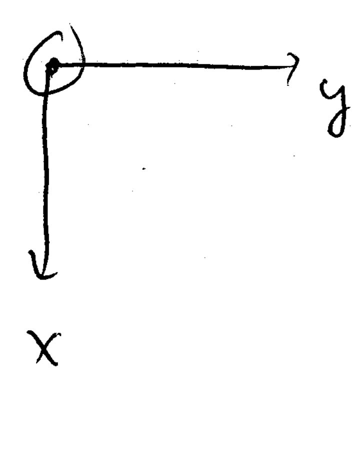
$$I(\text{bar}) = \frac{\mathcal{E}}{60}$$

In the magnetic field, the bar experiences an upward magnetic force

$$\vec{F}^{(m)} = I(\text{bar})\left(L\hat{i}\right) \times \left(B\hat{j}\right)$$

$$= \frac{\mathcal{E}}{60}(1.5\,\text{m})(2.0\,\text{T})\,\hat{k}$$

$$= \frac{\mathcal{E}}{20}\,\hat{k}$$



The gravitational force on the metal bar is downward, and has a magnitude of

$$|\vec{F}^{(G)}| = mg$$

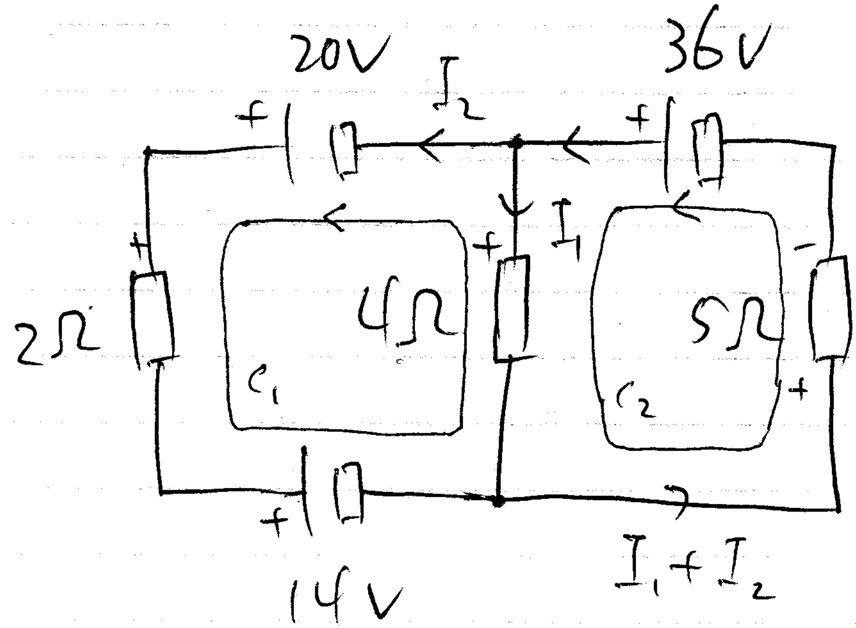
In order to have the upward magnetic force just balances the gravitational force, we need $\mathcal{E}$ such that

$$\frac{\mathcal{E}}{20} = mg = (0.5\,kg)(9.8\,m/s^2)$$

$$\therefore \mathcal{E} = 98 \text{ volts}$$

\#

5.



Assign two currents, I_1 and I_2, so that all currents satisfy the junction rule.

Along Loop C_1 and counter-clockwise:

$$2I_2 - 4I_1 + 14 - 20 = 0 \quad \cdots \quad (1)$$

$$\Rightarrow \quad I_2 - 2I_1 = 3 \quad \cdots \quad (1)'$$

Along Loop C_2 and also counter-clockwise:

$$4I_1 + 5(I_1 + I_2) - 36 = 0 \quad \cdots \quad (2)$$

$$\Rightarrow \quad 9I_1 + 5I_2 = 36 \quad \cdots \quad (2)'$$

$\cancel{(1)' \times 8 + (2)'}$ $(2)' - (1)' \times 5$: $19I_1 = 21 \quad \cdots \quad (3)$

$$\therefore \; I_1 = \frac{21}{19}\,A \quad \text{------} \quad (3)'$$

From $(1)'$:

$$I_2 = 3 + 2I_1 = \frac{99}{19}\,A \quad \text{-----} \quad (4)$$

And

$$I_1 + I_2 \Big|_{\text{through } 5\Omega} = \frac{120}{19}\,A \quad \text{--} \quad (5)$$

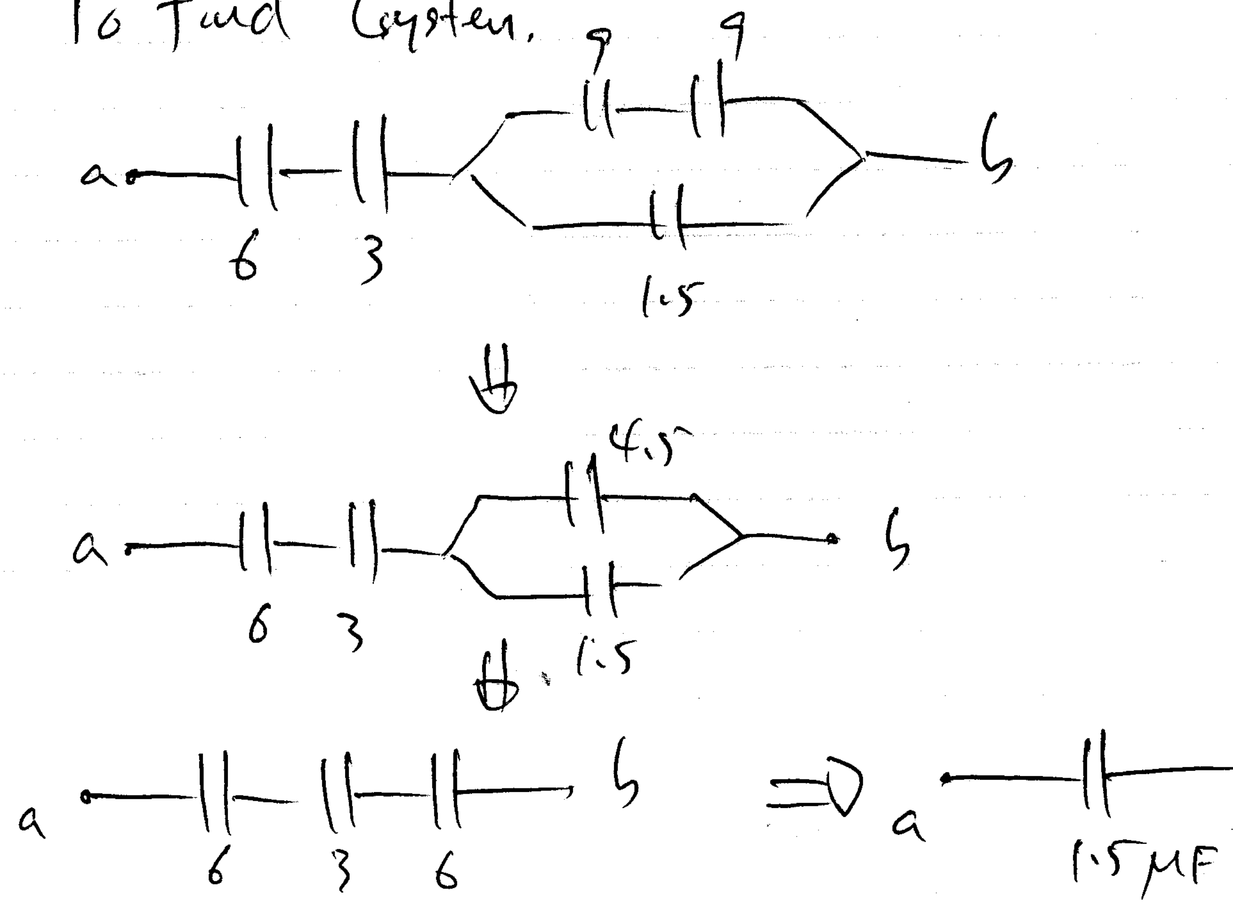
6-(a) Once we know the network capacitance C_{system}, the stored energy in the system (or network) when 12 volts is applied between a and b can be found as

$$U_{system} = \frac{C_{system}}{2} \cdot (12 \text{ volts})^2$$

$$= \frac{1}{2} C_{system} \cdot (\Delta V)_{ab}^2$$

$$= \frac{1}{2} C_{system} (V_a - V_b)^2$$

To find C_{system},



So,

$$C_{system} = 1.5\ \mu F$$

Thus

$$U_{system} = \frac{1}{2} C_{system} (V_a - V_b)^2$$

$$= \frac{1}{2} (1.5 \times 10^{-6} F)(12 V)^2$$

$$= 108\ \mu J$$

*

6-(b) To find the energy stored in the $3\mu F$ capacitor, we need to know the charge Q' (conveniently) or the potential drop $\Delta V_{3\mu F}$ across it.

By definition, Q' on $3\mu F$ is the same as the charge on C_{system} when $V_a - V_b = 12V$. Thus

$$Q'\Big|_{on\ 3\mu F} = C_{system}(V_a - V_b) = (1.5\mu F)(12V)$$

Thus

$$U_{3\mu F} = \frac{Q'^2}{2 \times (3 \times 10^{-6} F)} = \frac{1}{2} \frac{(1.5\mu F)^2 (12V)^2}{3 \times 10^{-6} F} = 54\ \mu J$$

*

Or, the potential drop across $3\mu F$ is one half of the potential difference $(V_a - V_b)$, namely, $\Delta V_{3\mu F} = 6\,V$. Thus

$$U_{3\mu F} = \frac{1}{2}(3\mu F)(\Delta V_{3\mu F})^2 = 54\,\mu J$$

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7-(a) The electric field produced by an infinite sheet with a circular hole is the difference of the electric field produced by an infinite sheet without a hole and ~~[illegible]~~ the electric field produced by a circular disc with the same diameter and same surface charge density.

Thus

$$\vec{E}(z)\Big|_{\substack{\text{sheet with}\\ \text{a hole}}} = \vec{E}(z)\Big|_{\substack{\text{sheet}\\ \text{without}\\ \text{a hole}}} - \vec{E}(z)\Big|_{\substack{\text{circular}\\ \text{disc}}}$$

$$= \frac{\sigma_0}{2\varepsilon_0}\hat{k} - \frac{\sigma_0}{2\varepsilon_0}\hat{k}\left(1 - \frac{z}{\sqrt{z^2+r^2}}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\hat{k}\cdot\frac{z}{\sqrt{z^2+r^2}}$$

$$= \frac{\sigma_0}{2\varepsilon_0}\frac{z}{(z^2+r^2)^{1/2}}\hat{k}$$

#

7-(b) By definition of the electric potential difference

$$V(0) - V(z) = \int_0^z \vec{E}(z') \cdot d\vec{l} \qquad d\vec{l} = \hat{k}\, dz'$$

$$= \frac{\sigma_0}{2\varepsilon_0} \int_0^z \frac{z' dz'}{(z'^2 + r^2)^{1/2}}$$

$$= \frac{\sigma_0}{2\varepsilon_0} \left(\sqrt{z^2 + r^2} - r \right)$$

$$\therefore\ V(z) - V(0) = -\frac{\sigma_0}{2\varepsilon_0} \left(\sqrt{z^2 + r^2} - r \right)$$

7-(c) Let z be the coordinate at which the electron comes to a stop. Then by energy conservation,

$$\frac{1}{2} m_e V_0^2 = q_e \left(V(z) - V(0) \right)$$

or

$$\frac{1}{2} m_e V_0^2 = \frac{e \sigma_0 r}{2\varepsilon_0} \left(\sqrt{1 + (z/r)^2} - 1 \right)$$

We have

$$\left(1+\left(z/r\right)^2\right)^{1/2} = 1 + \frac{m_e \epsilon_0 V_0^2}{e \cdot \sigma_0 r}$$

$$= 1 + \frac{9.1\times10^{-31} \times 8.85\times10^{-12} \times 2\times10^{7} \times 2\times10^{7}}{1.6\times10^{-19} \times 4\times10^{-7} \times 10^{-2}}$$

$$\doteq 6$$

$$\therefore z \simeq \sqrt{35} \cdot r = 5.9 \text{ cm}$$