Solution to 9C-A (2010) Midterm #2

1-(a) C_2 and C_3 are in series, and can be represented by an equivalent ~~[illegible]~~ capacitor C_{23}

$$C_{23} = \frac{1}{1/C_2 + 1/C_3} = \frac{C_2 C_3}{C_2 + C_3} = 4\mu F$$

C_1 and C_{23} are in parallel, and can be represented by an equivalent capacitor C_{123}

$$C_{123} = C_1 + C_{23} = 12\,\mu F$$

C_4 and C_{123} are in series, and can be represented by an equivalent capacitor C_{1234} (the one we are looking for)

$$C_{1234} = \frac{1}{1/C_4 + 1/C_{123}} = \frac{C_4 \cdot C_{123}}{C_4 + C_{123}} = 4.8\mu F$$

1-(b) The charge on C_4 is the same as the charge on the C_{1234}, thus

$$Q_4 = C_{1234} \cdot V_{ab} = 96\,\mu C$$

The potential difference $V_{ad} = V_a - V_d$ equals to the charge on C_{1234} divided by C_{123} as C_{123} is in series with C_4,

$$V_{ad} = \frac{Q_4}{C_{123}} = \frac{96\,\mu C}{12\,\mu F} = 8\,V$$

Thus the charge on C_1 is given by

$$Q_1 = C_1 \cdot V_{ad} = 64\,\mu C$$

The charges on C_2 and C_3 are the same, and given by

$$Q_2 = Q_3 = C_{23} \cdot V_{ad} = 32\,\mu C$$

1-(c) The potential difference across C_4 is

$$V_4 = Q_4/C_4 = 96\,\mu C/8\,\mu F = 12\,V$$

The potential difference across C_1 is

$$V_1 = Q_1/C_1 = 8\,V \quad (= V_{ab} - V_4)$$

The potential differences across C_2 and C_3 are the same, and equal to one half of V_1:

$$V_2 = V_3 = V_1/2 = 4\,V$$

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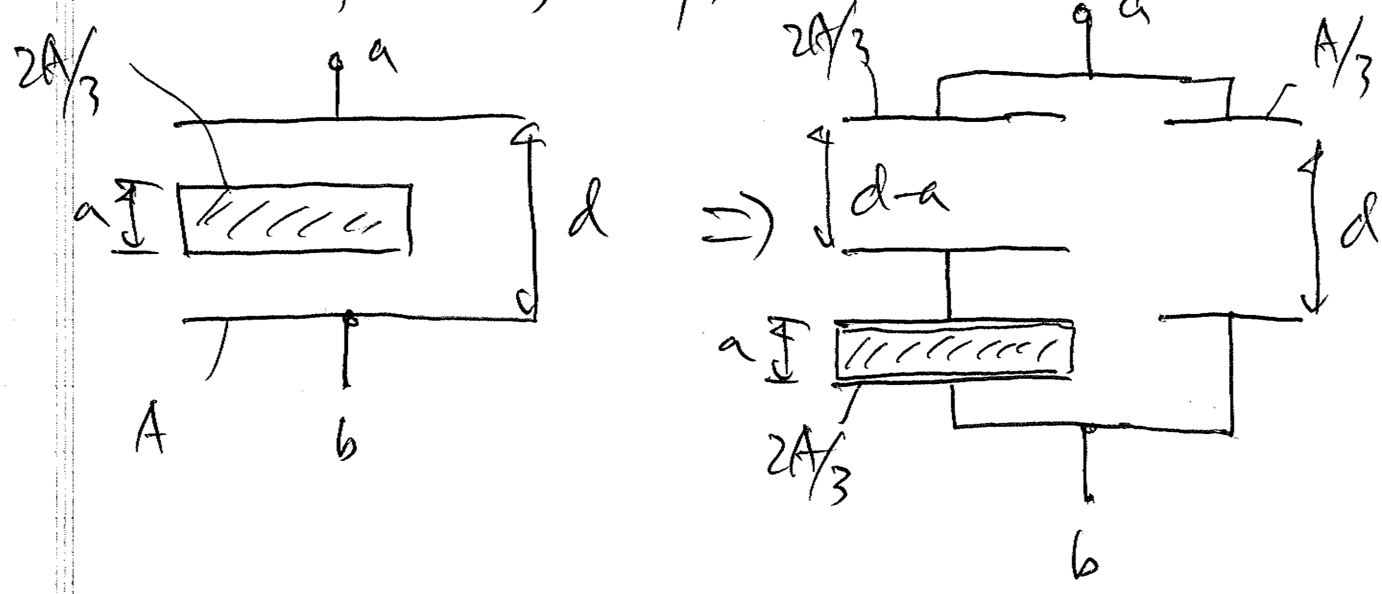
2-(a) With the dielectric slab inserted, the capacitor can be treated as 3 capacitors connected in the following way,



The equivalent capacitance of the left side is

$$C_L = \frac{\left(\epsilon_0 (2A/3)/(d-a)\right)\left(\epsilon_0 K (2A/3)/a\right)}{\epsilon_0(2A/3)/(d-a) + \epsilon_0 K (2A/3)/a}$$

$$= \frac{\epsilon_0(2A/3)}{d} \cdot \frac{K \cdot d}{a + K(d-a)}$$

The equivalent capacitance of the right side is

$$C_R = \frac{\epsilon_0 (A/3)}{d}$$

As a result,

$$C = C_L + C_R$$

$$= \frac{\epsilon_0 A}{d} \cdot \frac{(2/3)\,kd}{a + k(d-a)} + \frac{\epsilon_0 A}{d}\frac{1}{3}$$

$$= C_0 \cdot \left[\frac{(2/3)\,kd}{a + k(d-a)} + \frac{1}{3} \right]$$

$$= C_0 \cdot \left[\frac{4d}{a + 6(d-a)} + \frac{1}{3} \right]$$

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2-(b) When $a = 0$,

$$C = C_0$$

2-(c) When $a = d$,

$$C = C_0 \left(\frac{4d}{d} + \frac{1}{3} \right) = \left(4\tfrac{1}{3}\right) C_0.$$

3-(a) The equivalent resistance of R_2, R_3 and R_4 in parallel is

$$R_{234} = \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = \frac{R_1}{3} = 2\Omega.$$

The total resistance viewed from the two terminals of the emf is

$$R_{1234} = R_1 + R_{234} = 8\Omega \quad \text{(in series)}$$

The current through R_1 and R_{234} is the

$$I_1 = I_{234} = \frac{\mathcal{E}}{R_{1234}} = \frac{72V}{8\Omega} = 9A$$

The power dissipated in R_1 is

$$P_1 = I_1^2 \cdot R_1 = (9A)^2(6\Omega) = 486 \text{ Watt}.$$

The current through R_2 is $1/3$ of $I_{234} = 9A$, thus

$$P_2 = I_2^2 R_2 = \left(I_{234}/3\right)^2 \cdot R_2 = (3A)^2 6\Omega = 54 \text{ Watt}$$

3-(b) When R_4 is removed,

$$R_{23} = \frac{1}{1/R_2 + 1/R_3} = \frac{R_3}{2} = 3\Omega.$$

Thus the total resistance seen by the emf is

$$R_{123} = R_1 + R_{23} = 9\Omega.$$

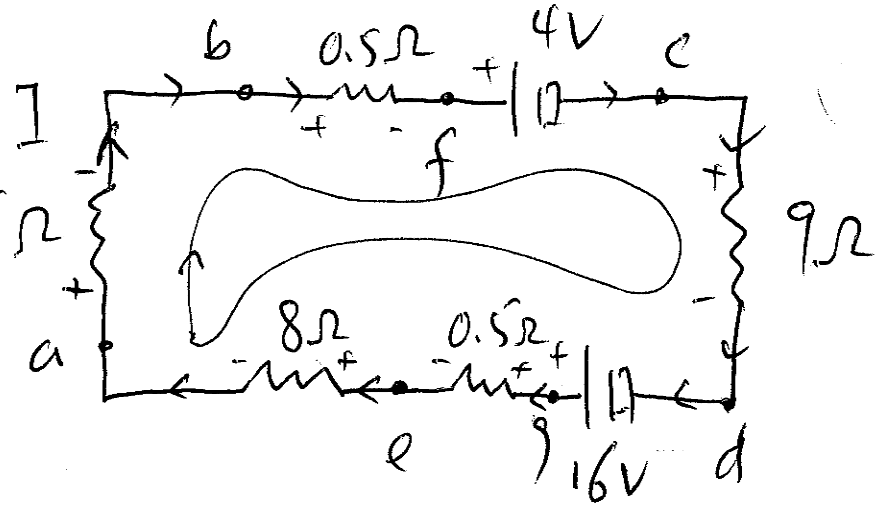
The current through R_1 and R_{23} is

$$I_1 = I_{23} = I_{123} = \frac{\varepsilon}{R_{123}} = \frac{72V}{9\Omega} = 8A.$$

$$P_1 = I_1^2 R_1 = (8A)^2 \cdot 6\Omega = 384 \text{ Watt}$$

$$P_2 = P_3 = \left(I_{23}/2\right)^2 R_2 = (4A)^2 R_2 = 96 \text{ Watt}.$$

4-(a) Assign the current in the clockwise direction,



Along the clockwise loop, starting from a, the potential drops add up to zero,

$$6I + 0.5I + 4 + 9I - 16 + 0.5I + 8I = 0$$

$\therefore\ 24I = 12V \qquad I = +0.5\,A.$

So the current I flows in the direction as assigned and has a magnitude of 0.5 A.

4-(b) The terminal voltage across the 4-V battery is

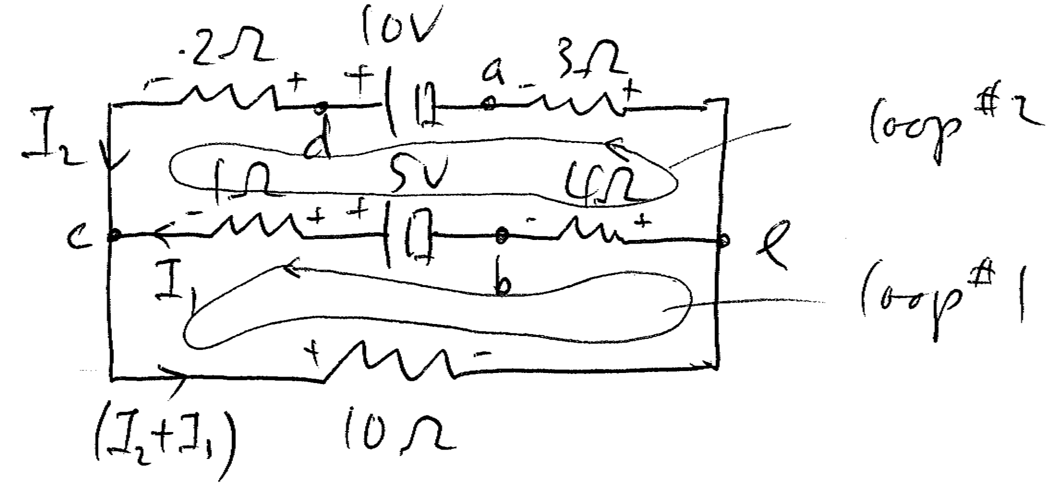
$$V_b - V_c = (V_b - V_f) + (V_f - V_c) = 0.5(0.5) + 4$$
$$= 4.25 \text{ Volts}$$

4-(c) $V_{ad} = V_a - V_d = (V_a - V_e) + (V_e - V_g) + (V_g - V_d)$

$$= -(0.5A)(8\Omega) - (0.5A)(0.5\Omega) + 16V$$

$$= 11.75V$$

5- (a) Assign two currents I_2 and I_1 in the direct-current circuit as follows



Along loop #1, start at point c, counterclockwise,

$$10(I_2+I_1) + 4I_1 - 5 + I_1 = 0 \quad \cdots (1)$$

$$3I_1 + 2I_2 = 1 \quad \cdots\cdots (1)'$$

Along loop #2, starting at point a, counterclockwise,

$$-10 + 2I_2 - I_1 + 5 - 4I_1 + 3I_2 = 0 \quad \cdots (2)$$

$$-I_1 + I_2 = 1 \quad \cdots\cdots (2)'$$

$$I_2 = 1 + I_1 \quad \cdots\cdots (2)''$$

Insert (2)'' into (1)':

$$5I_1 = -1 \qquad \therefore \; I_1 = -\tfrac{1}{5}\,A \quad \cdots (3)$$

$$I_2 = I_1 + 1A = +\,4/5\,A$$

$$(I_1 + I_2)\Big|_{\text{passing } 10\Omega} = +(3/5)A.$$

So, the current through 2Ω resister branch is in the direction as assigned, and has a magnitude of 0.8 A;

The current through 1Ω resister branch is in the oposite direction to the assigned, and has a magnitude of 0.2 A;

The current through 10Ω resister branch is in the direction as assigned, and has a magnitude of 0.6A.

5-(b)

$$\begin{aligned} V_{ab} = V_a - V_b &= (V_a - V_e) + (V_e - V_b) \\ &= -(3\Omega) I_2 + (4\Omega) I_1 \\ &= -(3\Omega)(0.8A) + (4\Omega)(-0.2A) \\ &= -3.2\,V \end{aligned}$$

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