21.56. **(a) IDENTIFY:** The potential energy is given by Eq. (21.17).

SET UP: $U(\phi) = -\vec{p} \cdot \vec{E} = -pE\cos\phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.

EXECUTE: parallel: $\phi = 0$ and $U(0°) = -pE$

perpendicular: $\phi = 90°$ and $U(90°) = 0$

$\Delta U = U(90°) - U(0°) = pE = (5.0 \times 10^{-30}\ \text{C} \cdot \text{m})(1.6 \times 10^{6}\ \text{N/C}) = 8.0 \times 10^{-24}\ \text{J}.$

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0 \times 10^{-24}\ \text{J})}{3(1.381 \times 10^{-23}\ \text{J/K})} = 0.39\ \text{K}$

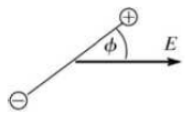
EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.57. **(a) IDENTIFY and SET UP:** Use Eq. (21.14) to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.

EXECUTE: $p = qd = (4.5 \times 10^{-9}\ \text{C})(3.1 \times 10^{-3}\ \text{m}) = 1.4 \times 10^{-11}\ \text{C} \cdot \text{m}$; The direction of $\vec{p}$ is from q_1 toward q_2.

(b) IDENTIFY and SET UP: Use Eq. (21.15) to relate the magnitudes of the torque and field.

EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.57, so



$$E = \frac{\tau}{p\sin\phi}$$

$$E = \frac{7.2 \times 10^{-9}\ \text{N} \cdot \text{m}}{(1.4 \times 10^{-11}\ \text{C} \cdot \text{m})\sin 36.9°} = 860\ \text{N/C}$$

Figure 21. 57

EVALUATE: Eq. (21.15) gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple (Problem 11.21) and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2 \times 10^{-8}\ \text{N} \cdot \text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

21.59. **IDENTIFY:** The torque on a dipole in an electric field is given by $\vec{\tau} = \vec{p} \times \vec{E}$.

SET UP: $\tau = pE\sin\phi$, where ϕ is the angle between the direction of $\vec{p}$ and the direction of $\vec{E}$.

EXECUTE: **(a)** The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* direction, as shown in Figure 21.59a.

(b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{p}$ back into alignment with $\vec{E}$. When $\vec{p}$ is directed opposite to $\vec{E}$, a small displacement results in a torque that takes $\vec{p}$ farther from alignment with $\vec{E}$.

(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.59b.

EVALUATE: The field of the dipole is directed from the + charge toward the – charge.

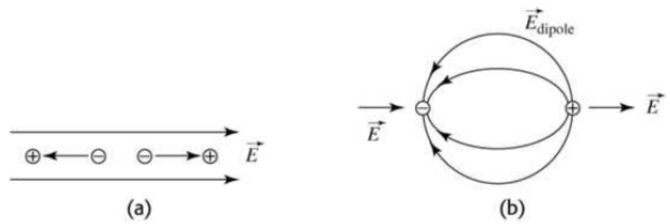


Figure 21. 59

21.104. **IDENTIFY:** Apply Eq. (21.11) for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk, Eq. (21.11) is

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - 1/\sqrt{(R/x)^2 + 1}\right].$$

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma$

(b) $\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2 + 1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2 + 1}\right]\right)\dfrac{|x|}{x}\hat{i}.$

$\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2 + 1} - 1/\sqrt{(R_2/x)^2 + 1}\right)\dfrac{|x|}{x}\hat{i}$. The electric field is in the $+x$-direction at points above the disk and in the $-x$-direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{i}$ specifies these directions.

(c) Note that $1/\sqrt{(R_1/x)^2 + 1} = \dfrac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \dfrac{|x|}{R_1}$. This gives

$\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\dfrac{|x|^2}{x}\hat{i} = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x\hat{i}$. Sufficiently close means that $(x/R_1)^2 \ll 1$.

(d) $F_x = -qE_x = -\dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x$. The force is in the form of Hooke's law: $F_x = -kx$, with

$k = \dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{q\sigma}{2\epsilon_0 m}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}$.

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

22.21. **IDENTIFY:** The electric field inside the conductor is zero, and all of its initial charge lies on its outer surface. The introduction of charge into the cavity induces charge onto the surface of the cavity, which induces an equal but opposite charge on the outer surface of the conductor. The net charge on the outer surface of the conductor is the sum of the positive charge initially there and the additional negative charge due to the introduction of the negative charge into the cavity.

(a) SET UP: First find the initial positive charge on the outer surface of the conductor using $q_i = \sigma A$, where A is the area of its outer surface. Then find the net charge on the surface after the negative charge has been introduced into the cavity. Finally, use the definition of surface charge density.

EXECUTE: The original positive charge on the outer surface is

$$q_i = \sigma A = \sigma(4\pi r^2) = (6.37\times10^{-6}\ \text{C/m}^2)4\pi(0.250\ \text{m})^2 = 5.00\times10^{-6}\ \text{C}$$

After the introduction of $-0.500\ \mu\text{C}$ into the cavity, the outer charge is now

$$5.00\ \mu\text{C} - 0.500\ \mu\text{C} = 4.50\ \mu\text{C}$$

The surface charge density is now $\sigma = \dfrac{q}{A} = \dfrac{q}{4\pi r^2} = \dfrac{4.50\times10^{-6}\ \text{C}}{4\pi(0.250\ \text{m})^2} = 5.73\times10^{-6}\ \text{C/m}^2$

(b) SET UP: Using Gauss's law, the electric field is $E = \dfrac{\Phi_E}{A} = \dfrac{q}{\epsilon_0 A} = \dfrac{q}{\epsilon_0 4\pi r^2}$.

EXECUTE: Substituting numbers gives

$$E = \frac{4.50\times10^{-6}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.250\ \text{m})^2} = 6.47\times10^5\ \text{N/C}.$$

(c) SET UP: We use Gauss's law again to find the flux. $\Phi_E = \dfrac{q}{\epsilon_0}.$

EXECUTE: Substituting numbers gives

$$\Phi_E = \frac{-0.500\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -5.65\times10^4\ \text{N}\cdot\text{m}^2/\text{C}.$$

EVALUATE: The excess charge on the conductor is still $+5.00\ \mu\text{C}$, as it originally was. The introduction of the $-0.500\ \mu\text{C}$ inside the cavity merely induced equal but opposite charges (for a net of zero) on the surfaces of the conductor.

22.32. **IDENTIFY:** The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma/2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A: $E_A = \dfrac{|\sigma_2|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_1|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|).$

$E_A = \dfrac{1}{2\epsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82\times10^5$ N/C to the left.

(b) $E_B = \dfrac{|\sigma_1|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|).$

$E_B = \dfrac{1}{2\epsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95\times10^5$ N/C to the left.

(c) $E_C = \dfrac{|\sigma_4|}{2\epsilon_0} + \dfrac{|\sigma_1|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} - \dfrac{|\sigma_3|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_4| + |\sigma_1| - |\sigma_2| - |\sigma_3|).$

$E_C = \dfrac{1}{2\epsilon_0}(4\ \mu\text{C/m}^2 + 6\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2 - 2\ \mu\text{C/m}^2) = 1.69\times10^5$ N/C to the left.

EVALUATE: The field at C is not zero. The pieces of plastic are not conductors.

22.34. **IDENTIFY:** Use Eq. (22.3) to calculate the flux for each surface. Use Eq. (22.8) to calculate the total enclosed charge.

SET UP: $\vec{E} = (-5.00 \text{ N/C}\cdot\text{m})x\hat{i} + (3.00 \text{ N/C}\cdot\text{m})z\hat{k}$. The area of each face is L^2, where $L = 0.300$ m.

EXECUTE: **(a)** $\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E}\cdot\hat{n}_{S_1}A = 0.$

$\hat{n}_{S_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E}\cdot\hat{n}_{S_2}A = (3.00 \text{ N/C}\cdot\text{m})(0.300 \text{ m})^2 z = (0.27 \text{ (N/C)}\cdot\text{m})z.$

$\Phi_2 = (0.27 \text{ (N/C)}\cdot\text{m})(0.300 \text{ m}) = 0.081 \text{(N/C)}\cdot\text{m}^2.$

$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E}\cdot\hat{n}_{S_3}A = 0.$

$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E}\cdot\hat{n}_{S_4}A = -(0.27 \text{ (N/C)}\cdot\text{m})z = 0$ (since $z = 0$).

$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E}\cdot\hat{n}_{S_5}A = (-5.00 \text{ N/C}\cdot\text{m})(0.300 \text{ m})^2 x = -(0.45 \text{ (N/C)}\cdot\text{m})x.$

$\Phi_5 = -(0.45 \text{ (N/C)}\cdot\text{m})(0.300 \text{ m}) = -(0.135 \text{(N/C)}\cdot\text{m}^2).$

$\hat{n}_{S_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E}\cdot\hat{n}_{S_6}A = +(0.45 \text{ (N/C)}\cdot\text{m})x = 0$ (since $x = 0$).

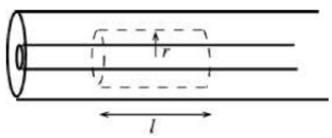
(b) Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)\text{(N/C)}\cdot\text{m}^2 = -0.054 \text{ N}\cdot\text{m}^2\text{/C}$. Therefore,

$q = \epsilon_0\Phi = -4.78\times10^{-13}$ C.

EVALUATE: Flux is positive when $\vec{E}$ is directed out of the volume and negative when it is directed into the volume.

22.39. **(a) IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $a < r < b$, and calculate E on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.39a.



EXECUTE: $\Phi_E = E(2\pi rl)$

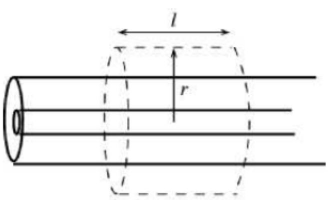
$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface).

Figure 22.39a

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(b) SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.39b.



EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

Figure 22.39b

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(b) SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.39b.

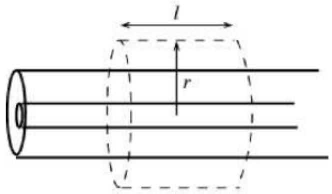


Figure 22.39b

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{encl} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$\Phi_E = \dfrac{Q_{encl}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(c) $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $a < r < b$; $E = 0$ for $b < r < c$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r > c$. The graph of E versus r is sketched in Figure 22.39c.

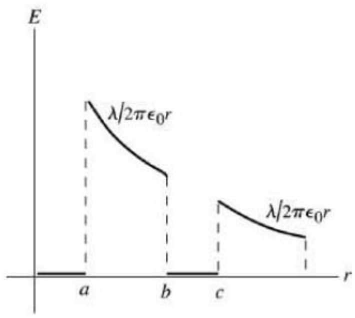


Figure 22.39c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

(d) IDENTIFY and SET UP: inner surface: Apply Gauss's law to a Gaussian cylinder with radius r, where $b < r < c$. We know E on this surface; calculate Q_{encl}.

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{encl} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

outer surface: The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per unit length $+\lambda$ on the outer surface.

EVALUATE: The electric field lines between the conductors originate on the surface charge on the outer surface of the inner conductor and terminate on the surface charges on the inner surface of the outer conductor. These surface charges are equal in magnitude (per unit length) and opposite in sign. The electric field lines outside the outer conductor originate from the surface charge on the outer surface of the outer conductor.

22.42. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l, and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi rl$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho \pi R^2$.

EXECUTE: **(a)** For $r < R$, $Q_{\text{encl}} = \rho \pi r^2 l$ and Gauss's law gives $E(2\pi rl) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\rho \pi r^2 l}{\epsilon_0}$ and $E = \frac{\rho r}{2\epsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho \pi R^2 l$ and Gauss's law gives $E(2\pi rl) = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{\rho \pi R^2 l}{\epsilon_0}$ and

$E = \frac{\rho R^2}{2\epsilon_0 r} = \frac{\lambda}{2\pi \epsilon_0 r}$, radially outward.

(c) At $r = R$, the electric field for BOTH regions is $E = \frac{\rho R}{2\epsilon_0}$, so they are consistent.

(d) The graph of E versus r is sketched in Figure 22.42.

EVALUATE: For $r > R$ the field is the same as for a line of charge along the axis of the cylinder.

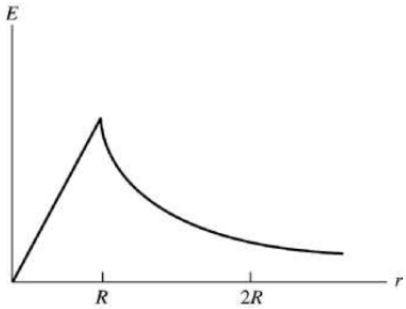


Figure 22.42

22.44. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the conducting spheres.

EXECUTE: **(a)** For $r < a$, $E = 0$, since these points are within the conducting material.

For $a < r < b$, $E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2}$, since there is $+q$ inside a radius r.

For $b < r < c$, $E = 0$, since these points are within the conducting material.

For $r > c$, $E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2}$, since again the total charge enclosed is $+q$.

(b) The graph of E versus r is sketched in Figure 22.44a.

(c) Since the Gaussian sphere of radius r, for $b < r < c$, must enclose zero net charge, the charge on the inner shell surface is $-q$.

(d) Since the hollow sphere has no net charge and has charge $-q$ on its inner surface, the charge on the outer shell surface is $+q$.

(e) The field lines are sketched in Figure 22.44b. Where the field is nonzero, it is radially outward.

EVALUATE: The net charge on the inner solid conducting sphere is on the surface of that sphere. The presence of the hollow sphere does not affect the electric field in the region $r < b$.

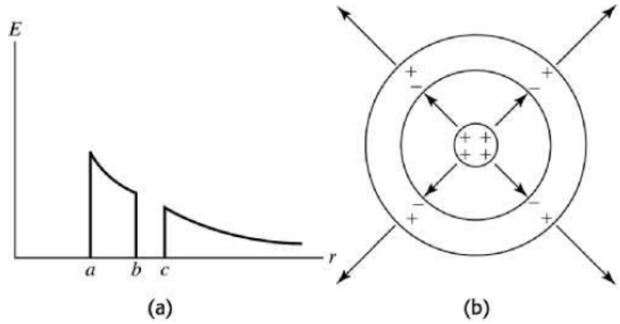


Figure 22.44

22.50. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the sphere and shell. The volume of the insulating shell is $V = \frac{4}{3}\pi([2R]^3 - R^3) = \frac{28\pi}{3}R^3$.

EXECUTE: **(a)** Zero net charge requires that $-Q = \frac{28\pi\rho R^3}{3}$, so $\rho = -\frac{3Q}{28\pi R^3}$.

(b) For $r < R, E = 0$ since this region is within the conducting sphere. For $r > 2R, E = 0$, since the net charge enclosed by the Gaussian surface with this radius is zero. For $R < r < 2R$, Gauss's law gives $E(4\pi r^2) = \frac{Q}{\epsilon_0} + \frac{4\pi\rho}{3\epsilon_0}(r^3 - R^3)$ and $E = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{\rho}{3\epsilon_0 r^2}(r^3 - R^3)$. Substituting ρ from part (a) gives $E = \frac{2}{7\pi\epsilon_0}\frac{Q}{r^2} - \frac{Qr}{28\pi\epsilon_0 R^3}$. The net enclosed charge for each r in this range is positive and the electric field is outward.

(c) The graph is sketched in Figure 22.50. We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere. But we see a smooth transition from the uniform insulator to the surrounding space.

EVALUATE: The expression for E within the insulator gives $E = 0$ at $r = 2R$.

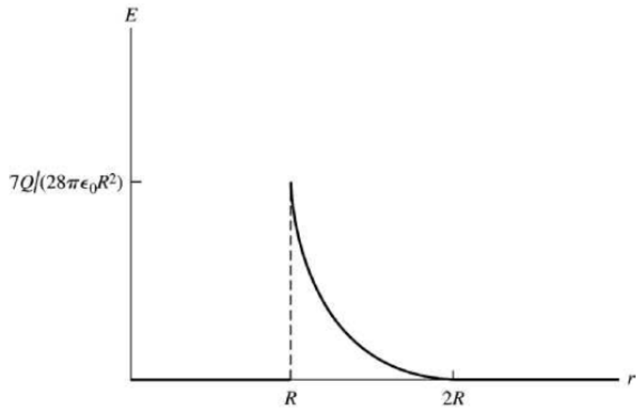
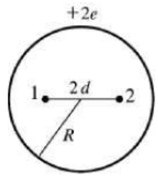


Figure 22.50

22.55. **IDENTIFY:** There is a force on each electron due to the other electron and a force due to the sphere of charge. Use Coulomb's law for the force between the electrons. Example 22.9 gives E inside a uniform sphere and Eq. (21.3) gives the force.

SET UP: The positions of the electrons are sketched in Figure 22.55a.



If the electrons are in equilibrium the net force on each one is zero.

Figure 22.55a

EXECUTE: Consider the forces on electron 2. There is a repulsive force F_1 due to the other electron, electron 1.

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(2d)^2}$$

The electric field inside the uniform distribution of positive charge is $E = \frac{Qr}{4\pi\epsilon_0 R^3}$ (Example 22.9), where $Q = +2e$. At the position of electron 2, $r = d$. The force F_{cd} exerted by the positive charge distribution is $F_{cd} = eE = \frac{e(2e)d}{4\pi\epsilon_0 R^3}$ and is attractive.

The force diagram for electron 2 is given in Figure 22.55b.

Figure 22.55b

Net force equals zero implies $F_1 = F_{cd}$ and $\frac{1}{4\pi\epsilon_0}\frac{e^2}{4d^2} = \frac{2e^2 d}{4\pi\epsilon_0 R^3}$.

Thus $(1/4d^2) = 2d/R^3$, so $d^3 = R^3/8$ and $d = R/2$.

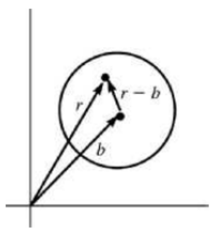
EVALUATE: The electric field of the sphere is radially outward; it is zero at the center of the sphere and increases with distance from the center. The force this field exerts on one of the electrons is radially inward and increases as the electron is farther from the center. The force from the other electron is radially outward, is infinite when $d = 0$ and decreases as d increases. It is reasonable therefore for there to be a value of d for which these forces balance.

22.61. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr'/4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}\,'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q/4\pi R^3$ so this may be written as $E = \rho r/3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}\,'$, so $\vec{E} = \rho\vec{r}\,'/3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}\,' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.61.



EXECUTE: Thus $\vec{E} = \frac{\rho(\vec{r} - \vec{b})}{3\epsilon_0}$

Figure 22.61

EVALUATE: When $b=0$ this reduces to the result of Example 22.9. When $\vec{r}=\vec{b}$, this gives $E=0$, which is correct since we know that $E=0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r}=\vec{b}$.

SET UP: $\vec{E}=\vec{E}_{\text{uniform}}+\vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho-\rho=0$.)

EXECUTE: $\vec{E}_{\text{uniform}}=\dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}}=\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}$, at points inside the hole.

Then $\vec{E}=\dfrac{\rho\vec{r}}{3\epsilon_0}+\left(\dfrac{-\rho(\vec{r}-\vec{b})}{3\epsilon_0}\right)=\dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.62. **IDENTIFY:** We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.

SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}\,'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.62, $\vec{r}\,'=\vec{r}-\vec{b}$. Problem 22.42 shows that at points within a long insulating cylinder, $\vec{E}=\dfrac{\rho\vec{r}}{2\epsilon_0}$.

EXECUTE: $\vec{E}_{\text{off-axis}}=\dfrac{\rho\vec{r}\,'}{2\epsilon_0}=\dfrac{\rho(\vec{r}-\vec{b})}{2\epsilon_0}$. $\vec{E}_{\text{hole}}=\vec{E}_{\text{cylinder}}-\vec{E}_{\text{off-axis}}=\dfrac{\rho\vec{r}}{2\epsilon_0}-\dfrac{\rho(\vec{r}-\vec{b})}{2\epsilon_0}=\dfrac{\rho\vec{b}}{2\epsilon_0}$.

Note that $\vec{E}$ is uniform.

EVALUATE: If the hole is coaxial with the cylinder, $b=0$ and $E_{\text{hole}}=0$.

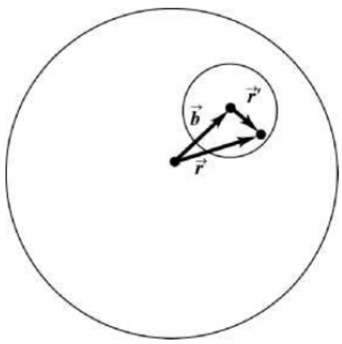


Figure 22.62