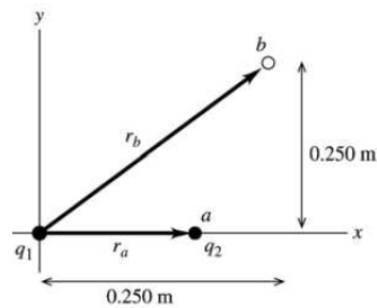


- 23.1. IDENTIFY:** Apply Eq. (23.2) to calculate the work. The electric potential energy of a pair of point charges is given by Eq. (23.9).  
**SET UP:** Let the initial position of  $q_2$  be point  $a$  and the final position be point  $b$ , as shown in Figure 23.1.



$$r_a = 0.150 \text{ m}$$

$$r_b = \sqrt{(0.250 \text{ m})^2 + (0.250 \text{ m})^2}$$

$$r_b = 0.3536 \text{ m}$$

**Figure 23.1**

**EXECUTE:**  $W_{a \rightarrow b} = U_a - U_b$

$$U_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_a} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(+2.40 \times 10^{-6} \text{ C})(-4.30 \times 10^{-6} \text{ C})}{0.150 \text{ m}}$$

$$U_a = -0.6184 \text{ J}$$

$$U_b = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_b} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(+2.40 \times 10^{-6} \text{ C})(-4.30 \times 10^{-6} \text{ C})}{0.3536 \text{ m}}$$

$$U_b = -0.2623 \text{ J}$$

$$W_{a \rightarrow b} = U_a - U_b = -0.6184 \text{ J} - (-0.2623 \text{ J}) = -0.356 \text{ J}$$

**EVALUATE:** The attractive force on  $q_2$  is toward the origin, so it does negative work on  $q_2$  when  $q_2$  moves to larger  $r$ .

- 23.3. IDENTIFY:** The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

**SET UP:** The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is  $U = (1/4\pi\epsilon_0)(qq_0/r)$ . Each charge is  $e$  and the charges are equidistant from each other,

$$\text{so the total potential energy is } U = \frac{1}{4\pi\epsilon_0} \left( \frac{e^2}{r} + \frac{e^2}{r} + \frac{e^2}{r} \right) = \frac{3e^2}{4\pi\epsilon_0 r}.$$

**EXECUTE:** Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60 \times 10^{-19} \text{ C})^2 (9.00 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)}{2.00 \times 10^{-15} \text{ m}} = 3.46 \times 10^{-13} \text{ J} = 2.16 \text{ MeV}$$

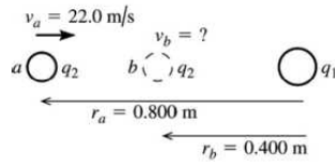
**EVALUATE:** This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

23.5. (a) **IDENTIFY:** Use conservation of energy:

$$K_a + U_a + W_{\text{other}} = K_b + U_b$$

$U$  for the pair of point charges is given by Eq. (23.9).

**SET UP:**



Let point  $a$  be where  $q_2$  is 0.800 m from  $q_1$  and point  $b$  be where  $q_2$  is 0.400 m from  $q_1$ , as shown in Figure 23.5a.

Figure 23.5a

**EXECUTE:** Only the electric force does work, so  $W_{\text{other}} = 0$  and  $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$ .

$$K_a = \frac{1}{2} m v_a^2 = \frac{1}{2} (1.50 \times 10^{-3} \text{ kg}) (22.0 \text{ m/s})^2 = 0.3630 \text{ J}$$

$$U_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_a} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{0.800 \text{ m}} = +0.2454 \text{ J}$$

$$K_b = \frac{1}{2} m v_b^2$$

$$U_b = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_b} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{0.400 \text{ m}} = +0.4907 \text{ J}$$

The conservation of energy equation then gives  $K_b = K_a + (U_a - U_b)$

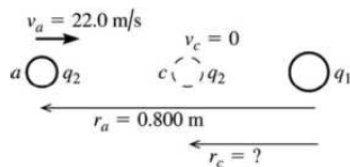
$$\frac{1}{2} m v_b^2 = +0.3630 \text{ J} + (0.2454 \text{ J} - 0.4907 \text{ J}) = 0.1177 \text{ J}$$

$$v_b = \sqrt{\frac{2(0.1177 \text{ J})}{1.50 \times 10^{-3} \text{ kg}}} = 12.5 \text{ m/s}$$

**EVALUATE:** The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) **IDENTIFY:** Let point  $c$  be where  $q_2$  has its speed momentarily reduced to zero. Apply conservation of energy to points  $a$  and  $c$ :  $K_a + U_a + W_{\text{other}} = K_c + U_c$ .

**SET UP:** Points  $a$  and  $c$  are shown in Figure 23.5b.



**EXECUTE:**  $K_a = +0.3630 \text{ J}$  (from part (a))

$U_a = +0.2454 \text{ J}$  (from part (a))

Figure 23.5b

$K_c = 0$  (at distance of closest approach the speed is zero)

$$U_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_c}$$

Thus conservation of energy  $K_a + U_a = U_c$  gives  $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_c} = +0.3630 \text{ J} + 0.2454 \text{ J} = 0.6084 \text{ J}$

$$r_c = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{0.6084 \text{ J}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(-2.80 \times 10^{-6} \text{ C})(-7.80 \times 10^{-6} \text{ C})}{+0.6084 \text{ J}} = 0.323 \text{ m}$$

**EVALUATE:**  $U \rightarrow \infty$  as  $r \rightarrow 0$  so  $q_2$  will stop no matter what its initial speed is.

**23.15. IDENTIFY:** Apply the equation that precedes Eq. (23.17):  $W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l}$ .

**SET UP:** Use coordinates where  $+y$  is upward and  $+x$  is to the right. Then  $\vec{E} = E\hat{j}$  with  $E = 4.00 \times 10^4$  N/C.

(a) The path is sketched in Figure 23.15a.

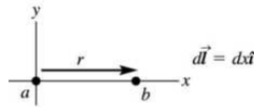


Figure 23.15a

**EXECUTE:**  $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i}) = 0$  so  $W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l} = 0$ .

**EVALUATE:** The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

(b) **SET UP:** The path is sketched in Figure 23.15b.

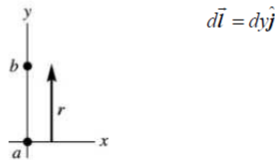


Figure 23.15b

**EXECUTE:**  $\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dy\hat{j}) = E dy$

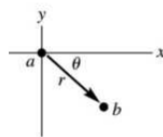
$$W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l} = q' E \int_a^b dy = q' E (y_b - y_a)$$

$y_b - y_a = +0.670$  m, positive since the displacement is upward and we have taken  $+y$  to be upward.

$$W_{a \rightarrow b} = q' E (y_b - y_a) = (+28.0 \times 10^{-9} \text{ C})(4.00 \times 10^4 \text{ N/C})(+0.670 \text{ m}) = +7.50 \times 10^{-4} \text{ J.}$$

**EVALUATE:** The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

(c) **SET UP:** The path is sketched in Figure 23.15c.



$$y_a = 0$$

$$y_b = -r \sin \theta = -(2.60 \text{ m}) \sin 45^\circ = -1.838 \text{ m}$$

The vertical component of the 2.60 m displacement is 1.838 m downward.

Figure 23.15c

**EXECUTE:**  $d\vec{l} = dx\hat{i} + dy\hat{j}$  (The displacement has both horizontal and vertical components.)

$\vec{E} \cdot d\vec{l} = (E\hat{j}) \cdot (dx\hat{i} + dy\hat{j}) = E dy$  (Only the vertical component of the displacement contributes to the work.)

$$W_{a \rightarrow b} = q' \int_a^b \vec{E} \cdot d\vec{l} = q' E \int_a^b dy = q' E (y_b - y_a)$$

$$W_{a \rightarrow b} = q' E (y_b - y_a) = (+28.0 \times 10^{-9} \text{ C})(4.00 \times 10^4 \text{ N/C})(-1.838 \text{ m}) = -2.06 \times 10^{-3} \text{ J.}$$

**EVALUATE:** The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

23.19. IDENTIFY:  $V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$

SET UP: The locations of the charges and points  $A$  and  $B$  are sketched in Figure 23.19.

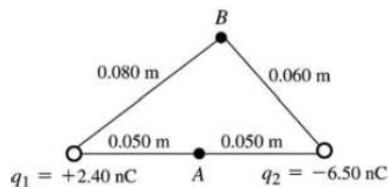


Figure 23.19

EXECUTE: (a)  $V_A = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_{A1}} + \frac{q_2}{r_{A2}} \right)$

$$V_A = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{+2.40 \times 10^{-9} \text{ C}}{0.050 \text{ m}} + \frac{-6.50 \times 10^{-9} \text{ C}}{0.050 \text{ m}} \right) = -737 \text{ V}$$

(b)  $V_B = \frac{1}{4\pi\epsilon_0} \left( \frac{q_1}{r_{B1}} + \frac{q_2}{r_{B2}} \right)$

$$V_B = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left( \frac{+2.40 \times 10^{-9} \text{ C}}{0.080 \text{ m}} + \frac{-6.50 \times 10^{-9} \text{ C}}{0.060 \text{ m}} \right) = -704 \text{ V}$$

(c) IDENTIFY and SET UP: Use Eq. (23.13) and the results of parts (a) and (b) to calculate  $W$ .

EXECUTE:  $W_{B \rightarrow A} = q'(V_B - V_A) = (2.50 \times 10^{-9} \text{ C})(-704 \text{ V} - (-737 \text{ V})) = +8.2 \times 10^{-8} \text{ J}$

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point  $B$ ) to lower potential (point  $A$ ).

23.21. IDENTIFY: For a point charge,  $V = \frac{kq}{r}$ . The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: (a) The positions of the two charges are shown in Figure 23.21a.

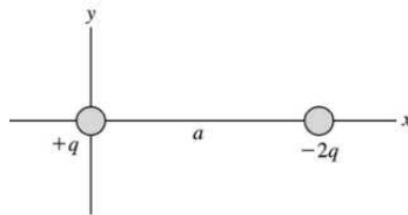


Figure 23.21a

(b)  $x > a: V = \frac{kq}{x} - \frac{2kq}{x-a} = \frac{-kq(x+a)}{x(x-a)}$ .  $0 < x < a: V = \frac{kq}{x} - \frac{2kq}{a-x} = \frac{kq(3x-a)}{x(x-a)}$ .

$x < 0: V = \frac{-kq}{x} + \frac{2kq}{x-a} = \frac{kq(x+a)}{x(x-a)}$ . A general expression valid for any  $y$  is  $V = k \left( \frac{q}{|x|} - \frac{2q}{|x-a|} \right)$ .

(c) The potential is zero at  $x = -a$  and  $a/3$ .

(d) The graph of  $V$  versus  $x$  is sketched in Figure 23.21b.

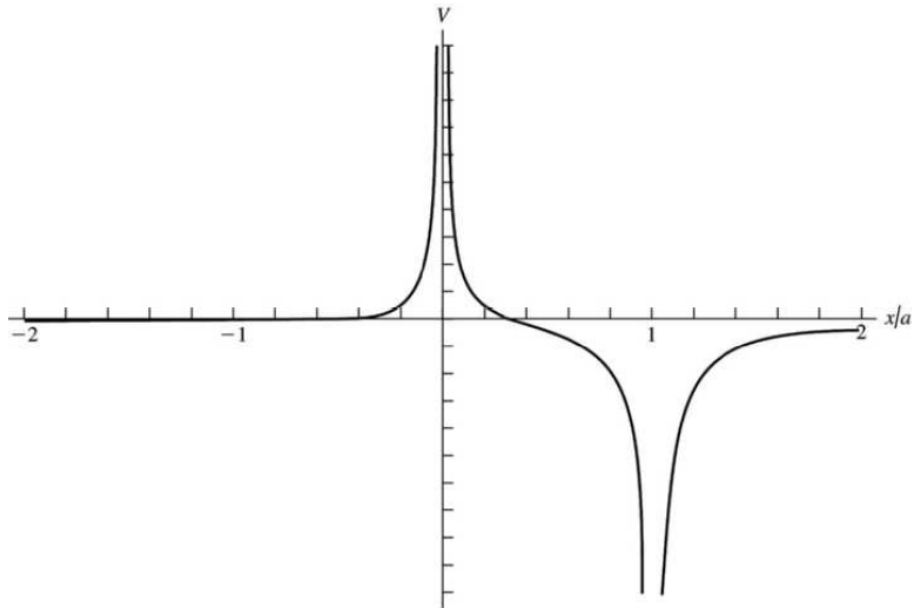


Figure 23.21 b

**EVALUATE:** (e) For  $x \gg a$ :  $V \approx \frac{-kqx}{x^2} = \frac{-kq}{x}$ , which is the same as the potential of a point charge  $-q$ .

Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

- 23.25. (a) **IDENTIFY** and **SET UP:** The direction of  $\vec{E}$  is always from high potential to low potential so point  $b$  is at higher potential.

(b) Apply Eq. (23.17) to relate  $V_b - V_a$  to  $E$ .

**EXECUTE:**  $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E dx = E(x_b - x_a)$ .

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240 \text{ V}}{0.90 \text{ m} - 0.60 \text{ m}} = 800 \text{ V/m}$$

(c)  $W_{b \rightarrow a} = q(V_b - V_a) = (-0.200 \times 10^{-6} \text{ C})(+240 \text{ V}) = -4.80 \times 10^{-5} \text{ J}$ .

**EVALUATE:** The electric force does negative work on a negative charge when the negative charge moves from high potential (point  $b$ ) to low potential (point  $a$ ).

- 23.29. (a) **IDENTIFY** and **SET UP:** The electric field on the ring's axis is calculated in Example 21.9. The force on the electron exerted by this field is given by Eq. (21.3).

**EXECUTE:** When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form  $F = -kx$  so the oscillatory motion is not simple harmonic motion.

(b) **IDENTIFY:** Apply conservation of energy to the motion of the electron.

**SET UP:**  $K_a + U_a = K_b + U_b$  with  $a$  at the initial position of the electron and  $b$  at the center of the ring.

From Example 23.11,  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2 + R^2}}$ , where  $R$  is the radius of the ring.

**EXECUTE:**  $x_a = 30.0 \text{ cm}$ ,  $x_b = 0$ .

$K_a = 0$  (released from rest),  $K_b = \frac{1}{2}mv^2$

Thus  $\frac{1}{2}mv^2 = U_a - U_b$

And  $U = qV = -eV$  so  $v = \sqrt{\frac{2e(V_b - V_a)}{m}}$ .

$$V_a = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x_a^2 + R^2}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{24.0 \times 10^{-9} \text{ C}}{\sqrt{(0.300 \text{ m})^2 + (0.150 \text{ m})^2}}$$

$$V_a = 643 \text{ V}$$

$$V_b = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x_b^2 + R^2}} = (8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{24.0 \times 10^{-9} \text{ C}}{0.150 \text{ m}} = 1438 \text{ V}$$

$$v = \sqrt{\frac{2e(V_b - V_a)}{m}} = \sqrt{\frac{2(1.602 \times 10^{-19} \text{ C})(1438 \text{ V} - 643 \text{ V})}{9.109 \times 10^{-31} \text{ kg}}} = 1.67 \times 10^7 \text{ m/s}$$

**EVALUATE:** The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

- 23.38. IDENTIFY and SET UP:** For oppositely charged parallel plates,  $E = \sigma/\epsilon_0$  between the plates and the potential difference between the plates is  $V = Ed$ .

**EXECUTE:** (a)  $E = \frac{\sigma}{\epsilon_0} = \frac{47.0 \times 10^{-9} \text{ C/m}^2}{\epsilon_0} = 5310 \text{ N/C}$ .

(b)  $V = Ed = (5310 \text{ N/C})(0.0220 \text{ m}) = 117 \text{ V}$ .

(c) The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

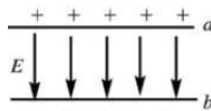
**EVALUATE:** The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles, the work, which is force times distance, doubles and the potential difference doubles.

- 23.39. IDENTIFY and SET UP:** Use the result of Example 23.9 to relate the electric field between the plates to the potential difference between them and their separation. The force this field exerts on the particle is given by Eq. (21.3). Use the equation that precedes Eq. (23.17) to calculate the work.

**EXECUTE:** (a) From Example 23.9,  $E = \frac{V_{ab}}{d} = \frac{360 \text{ V}}{0.0450 \text{ m}} = 8000 \text{ V/m}$ .

(b)  $F = |q|E = (2.40 \times 10^{-9} \text{ C})(8000 \text{ V/m}) = +1.92 \times 10^{-5} \text{ N}$

(c) The electric field between the plates is shown in Figure 23.39.



**Figure 22.39**

The plate with positive charge (plate  $a$ ) is at higher potential. The electric field is directed from high potential toward low potential (or,  $\vec{E}$  is from  $+$  charge toward  $-$  charge), so  $\vec{E}$  points from  $a$  to  $b$ . Hence the force that  $\vec{E}$  exerts on the positive charge is from  $a$  to  $b$ , so it does positive work.

$$W = \int_a^b \vec{F} \cdot d\vec{l} = Fd, \text{ where } d \text{ is the separation between the plates.}$$

$$W = Fd = (1.92 \times 10^{-5} \text{ N})(0.0450 \text{ m}) = +8.64 \times 10^{-7} \text{ J}$$

(d)  $V_a - V_b = +360 \text{ V}$  (plate  $a$  is at higher potential)

$$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40 \times 10^{-9} \text{ C})(-360 \text{ V}) = -8.64 \times 10^{-7} \text{ J.}$$

- 23.56. **IDENTIFY:** The electric force between the electron and proton is attractive and has magnitude  $F = \frac{ke^2}{r^2}$ .

For circular motion the acceleration is  $a_{\text{rad}} = v^2/r$ .  $U = -k\frac{e^2}{r}$ .

**SET UP:**  $e = 1.60 \times 10^{-19}$  C.  $1 \text{ eV} = 1.60 \times 10^{-19}$  J.

**EXECUTE:** (a)  $\frac{mv^2}{r} = \frac{ke^2}{r^2}$  and  $v = \sqrt{\frac{ke^2}{mr}}$ .

(b)  $K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{ke^2}{r} = -\frac{1}{2}U$

(c)  $E = K + U = \frac{1}{2}U = -\frac{1}{2} \frac{ke^2}{r} = -\frac{1}{2} \frac{k(1.60 \times 10^{-19} \text{ C})^2}{5.29 \times 10^{-11} \text{ m}} = -2.17 \times 10^{-18} \text{ J} = -13.6 \text{ eV}$ .

**EVALUATE:** The total energy is negative, so the electron is bound to the proton. Work must be done on the electron to take it far from the proton.

- 23.62. **IDENTIFY:** Apply  $\Sigma F_x = 0$  and  $\Sigma F_y = 0$  to the sphere. The electric force on the sphere is  $F_e = qE$ . The potential difference between the plates is  $V = Ed$ .

**SET UP:** The free-body diagram for the sphere is given in Figure 23.62.

**EXECUTE:**  $T \cos \theta = mg$  and  $T \sin \theta = F_e$  gives

$$F_e = mg \tan \theta = (1.50 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2) \tan(30^\circ) = 0.0085 \text{ N}.$$

$$F_e = Eq = \frac{Vq}{d} \text{ and } V = \frac{Fd}{q} = \frac{(0.0085 \text{ N})(0.0500 \text{ m})}{8.90 \times 10^{-6} \text{ C}} = 47.8 \text{ V}.$$

**EVALUATE:**  $E = V/d = 956 \text{ V/m}$ .  $E = \sigma/\epsilon_0$  and  $\sigma = E\epsilon_0 = 8.46 \times 10^{-9} \text{ C/m}^2$ .

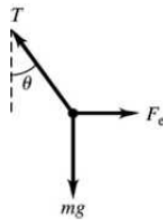


Figure 23.62

- 23.63. (a) **IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.
- SET UP:** From Example 23.10, for a conducting cylinder with charge per unit length  $\lambda$  the potential outside the cylinder is given by  $V = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$  where  $r$  is the distance from the cylinder axis and  $r_0$  is the distance from the axis for which we take  $V = 0$ . Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for  $V$  applies to both. This problem says to take  $r_0 = b$ .
- EXECUTE:** For the hollow tube of radius  $b$  and charge per unit length  $-\lambda$ : outside  $V = -(\lambda/2\pi\epsilon_0)\ln(b/r)$ ; inside  $V = 0$  since  $V = 0$  at  $r = b$ .
- For the metal cylinder of radius  $a$  and charge per unit length  $\lambda$ : outside  $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$ , inside  $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$ , the value at  $r = a$ .
- (i)  $r < a$ ; inside both  $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$
- (ii)  $a < r < b$ ; outside cylinder, inside tube  $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$
- (iii)  $r > b$ ; outside both the potentials are equal in magnitude and opposite in sign so  $V = 0$ .
- (b) For  $r = a$ ,  $V_a = (\lambda/2\pi\epsilon_0)\ln(b/a)$ .
- For  $r = b$ ,  $V_b = 0$ .

Thus  $V_{ab} = V_a - V_b = (\lambda/2\pi\epsilon_0)\ln(b/a)$ .

(c) **IDENTIFY and SET UP:** Use Eq. (23.23) to calculate  $E$ .

**EXECUTE:** 
$$E = -\frac{\partial V}{\partial r} = -\frac{\lambda}{2\pi\epsilon_0} \frac{\partial}{\partial r} \ln\left(\frac{b}{r}\right) = -\frac{\lambda}{2\pi\epsilon_0} \left(\frac{r}{b}\right) \left(-\frac{b}{r^2}\right) = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r}.$$

(d) The electric field between the cylinders is due only to the inner cylinder, so  $V_{ab}$  is not changed,

$$V_{ab} = (\lambda/2\pi\epsilon_0)\ln(b/a).$$

**EVALUATE:** The electric field is not uniform between the cylinders, so  $V_{ab} \neq E(b-a)$ .

**23.71. IDENTIFY:** We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

**SET UP:** If  $\rho$  is the uniform volume charge density, the charge of a spherical shell of radius  $r$  and thickness  $dr$  is  $dq = \rho 4\pi r^2 dr$ , and  $\rho = Q/(4/3 \pi R^3)$ . The charge already present in a sphere of radius  $r$  is  $q = \rho(4/3 \pi r^3)$ . The energy to bring the charge  $dq$  to the surface of the charge  $q$  is  $Vdq$ , where  $V$  is the potential due to  $q$ , which is  $q/4\pi\epsilon_0 r$ .

**EXECUTE:** The total energy to assemble the entire sphere of radius  $R$  and charge  $Q$  is sum (integral) of the tiny increments of energy.

$$U = \int Vdq = \int \frac{q}{4\pi\epsilon_0 r} dq = \int_0^R \frac{\rho \frac{4}{3} \pi r^3}{4\pi\epsilon_0 r} (\rho 4\pi r^2 dr) = \frac{3}{5} \left( \frac{1}{4\pi\epsilon_0} \frac{Q^2}{R} \right)$$

where we have substituted  $\rho = Q/(4/3 \pi R^3)$  and simplified the result.

**EVALUATE:** For a point charge,  $R \rightarrow 0$  so  $U \rightarrow \infty$ , which means that a point charge should have infinite self-energy. This suggests that either point charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

**23.78. IDENTIFY:**  $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$

**SET UP:**  $\vec{E}$  is radially outward, so  $\vec{E} \cdot d\vec{l} = E dr$ . Problem 22.44 shows that  $E(r) = 0$  for  $r \leq a$ ,

$$E(r) = kq/r^2 \text{ for } a < r < b, \quad E(r) = 0 \text{ for } b < r < c \text{ and } E(r) = kq/r^2 \text{ for } r > c.$$

**EXECUTE: (a)** At  $r = c$ :  $V_c = -\int_{\infty}^c \frac{kq}{r^2} dr = \frac{kq}{c}$ .

**(b)** At  $r = b$ :  $V_b = -\int_{\infty}^c \vec{E} \cdot d\vec{r} - \int_c^b \vec{E} \cdot d\vec{r} = \frac{kq}{c} - 0 = \frac{kq}{c}$ .

**(c)** At  $r = a$ :  $V_a = -\int_{\infty}^c \vec{E} \cdot d\vec{r} - \int_c^b \vec{E} \cdot d\vec{r} - \int_b^a \vec{E} \cdot d\vec{r} = \frac{kq}{c} - kq \int_b^a \frac{dr}{r^2} = kq \left[ \frac{1}{c} - \frac{1}{b} + \frac{1}{a} \right]$

**(d)** At  $r = 0$ :  $V_0 = kq \left[ \frac{1}{c} - \frac{1}{b} + \frac{1}{a} \right]$  since it is inside a metal sphere, and thus at the same potential as its surface.

**EVALUATE:** The potential difference between the two conductors is  $V_a - V_b = kq \left[ \frac{1}{a} - \frac{1}{b} \right]$ .