29.4. **IDENTIFY** and **SET UP:** Apply the result derived in Exercise 29.3: $Q = NBA/R$. In the present exercise the flux changes from its maximum value of $\Phi_B = BA$ to zero, so this equation applies. R is the total resistance so here $R = 60.0\ \Omega + 45.0\ \Omega = 105.0\ \Omega$.

29.7. **IDENTIFY:** Calculate the flux through the loop and apply Faraday's law.
SET UP: To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance r from the wire, is $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** $B = \dfrac{\mu_0 i}{2\pi r}$, into the page.

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r} Ldr.$

(c) $\Phi_B = \int_a^b d\Phi_B = \dfrac{\mu_0 iL}{2\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 iL}{2\pi}\ln(b/a).$

(d) $|\varepsilon| = \dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 L}{2\pi}\ln(b/a)\dfrac{di}{dt}.$

(e) $|\varepsilon| = \dfrac{\mu_0(0.240\text{ m})}{2\pi}\ln(0.360/0.120)(9.60\text{ A/s}) = 5.06\times10^{-7}\text{ V}.$

EVALUATE: The induced emf is proportional to the rate at which the current in the long straight wire is changing

29.16. **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.
SET UP and **EXECUTE:** The magnetic field is outward through the round coil and is decreasing, so the magnetic field due to the induced current must also point outward to oppose this decrease. Therefore the induced current is counterclockwise.
EVALUATE: Careful! Lenz's law does not say that the induced current flows to oppose the magnetic flux. Instead it says that the current flows to oppose the *change* in flux.

29.18. **IDENTIFY:** Apply Lenz's law.
SET UP: The field of the induced current is directed to oppose the change in flux in the primary circuit.
EXECUTE: **(a)** The magnetic field in A is to the left and is increasing. The flux is increasing so the field due to the induced current in B is to the right. To produce magnetic field to the right, the induced current flows through R from right to left.
(b) The magnetic field in A is to the right and is decreasing. The flux is decreasing so the field due to the induced current in B is to the right. To produce magnetic field to the right the induced current flows through R from right to left.
(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.
EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.23. **IDENTIFY:** A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.
SET UP: The induced emf is $\varepsilon = vBL\sin\phi$, where ϕ is the angle between the velocity and the magnetic field.
EXECUTE: **(a)** $\varepsilon = vBL\sin\phi = (5.00\text{ m/s})(0.450\text{ T})(0.300\text{ m})(\sin 90^\circ) = 0.675\text{ V}$
(b) The positive charges are moved to end b, so b is at the higher potential.
(c) $E = V/L = (0.675\text{ V})/(0.300\text{ m}) = 2.25\text{ V/m}$. The direction of $\vec{E}$ is from b to a.
(d) The positive charges are pushed to b, so b has an excess of positive charge.
(e) (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.
EVALUATE: The motional emf is large enough to have noticeable effects in some cases.

29.27. **IDENTIFY and SET UP:** $\varepsilon = vBL$. Use Lenz's law to determine the direction of the induced current. The force F_{ext} required to maintain constant speed is equal and opposite to the force F_I that the magnetic field exerts on the rod because of the current in the rod.

EXECUTE: **(a)** $\varepsilon = vBL = (7.50 \text{ m/s})(0.800 \text{ T})(0.500 \text{ m}) = 3.00 \text{ V}$

(b) $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from b to a in the rod.

(c) $I = \frac{\varepsilon}{R} = \frac{3.00 \text{ V}}{1.50 \ \Omega} = 2.00 \text{ A}$. $F_I = ILB\sin\phi = (2.00 \text{ A})(0.500 \text{ m})(0.800 \text{ T})\sin 90° = 0.800 \text{ N}$. $\vec{F}_I$ is to the left. To keep the bar moving to the right at constant speed an external force with magnitude $F_{ext} = 0.800 \text{ N}$ and directed to the right must be applied to the bar.

(d) The rate at which work is done by the force F_{ext} is $F_{ext}v = (0.800 \text{ N})(7.50 \text{ m/s}) = 6.00 \text{ W}$. The rate at which thermal energy is developed in the circuit is $I^2R = (2.00 \text{ A})^2(1.50 \ \Omega) = 6.00 \text{ W}$. These two rates are equal, as is required by conservation of energy.

EVALUATE: The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

29.29. **IDENTIFY:** The motion of the bar due to the applied force causes a motional emf to be induced across the ends of the bar, which induces a current through the bar. The magnetic field exerts a force on the bar due to this current.

SET UP: The applied force is to the left and equal to $F_{applied} = F_B = ILB$. $\varepsilon = BvL$ and $I = \frac{\varepsilon}{R} = \frac{BvL}{R}$.

EXECUTE: **(a)** $\vec{B}$ out of page and Φ_B decreasing, so the field of the induced current is out of the page inside the loop and the induced current is counterclockwise.

(b) Combining $F_{applied} = F_B = ILB$ and $\varepsilon = BvL$, we have $I = \frac{\varepsilon}{R} = \frac{BvL}{R}$. $F_{applied} = \frac{vB^2L^2}{R}$. The rate at which this force does work is $P_{applied} = F_{applied}v = \frac{(vBL)^2}{R} = \frac{[(5.90 \text{ m/s})(0.650 \text{ T})(0.360 \text{ m})]^2}{45.0 \ \Omega} = 0.0424 \text{ W}$.

EVALUATE: The power is small because the magnetic force is usually small compared to everyday forces.

29.54. **IDENTIFY:** Apply Faraday's law.

SET UP: For rotation about the y-axis the situation is the same as in Examples 29.3 and 29.4 and we can apply the results from those examples.

EXECUTE: **(a)** Rotating about the y-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$\varepsilon_{max} = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}$.

(b) Rotating about the x-axis: $\frac{d\Phi_B}{dt} = 0$ and $\varepsilon = 0$.

(c) Rotating about the z-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$\varepsilon_{max} = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}$.

EVALUATE: The maximum emf is the same if the loop is rotated about an edge parallel to the z-axis as it is when it is rotated about the z-axis.

29.60. **IDENTIFY:** Apply Newton's second law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v. At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv. The induced emf is counterclockwise, so it opposes the voltage of the battery, ε.

EXECUTE: **(a)** The net current in the loop is $I = \dfrac{\varepsilon - BLv}{R}$. The acceleration of the bar is $a = \dfrac{F}{m} = \dfrac{ILB\sin(90°)}{m} = \dfrac{(\varepsilon - BLv)LB}{mR}$. To find $v(t)$, set $\dfrac{dv}{dt} = a = \dfrac{(\varepsilon - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$\int_0^v \dfrac{dv}{(\varepsilon - BLv)} = \int_0^t \dfrac{LB}{mR}dt \rightarrow v = \dfrac{\varepsilon}{BL}(1 - e^{-B^2L^2t/mR}) = (22\text{ m/s})(1 - e^{-t/15\text{ s}})$. The graph of v versus t is sketched in Figure 29.60. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \varepsilon/R = 2.4$ A, $F = ILB = 1.296$ N, and $a = F/m = 1.4\text{ m/s}^2$.

(c) When $v = 2.0$ m/s, $a = \dfrac{[12\text{ V} - (1.5\text{ T})(0.36\text{ m})(2.0\text{ m/s})](0.36\text{ m})(1.5\text{ T})}{(0.90\text{ kg})(5.0\ \Omega)} = 1.3\text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\dfrac{\varepsilon}{BL} = \dfrac{12\text{ V}}{(1.5\text{ T})(0.36\text{ m})} = 22$ m/s, which makes the acceleration zero.

EVALUATE: The current in the circuit is counterclockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

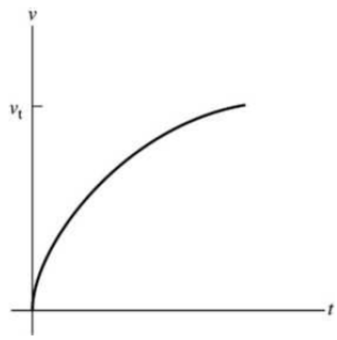


Figure 29.60

29.69. **(a) IDENTIFY:** Use Faraday's law to calculate the induced emf, Ohm's law to calculate I, and Eq. (27.19) to calculate the force on the rod due to the induced current.

SET UP: The force on the wire is shown in Figure 29.69.

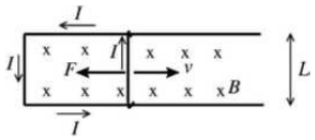


Figure 29.69

EXECUTE: When the wire has speed v the induced emf is $\varepsilon = BvL$ and the induced current is $I = \varepsilon/R = \dfrac{BvL}{R}$.

The induced current flows upward in the wire as shown, so the force $\vec{F} = I\vec{l} \times \vec{B}$ exerted by the magnetic field on the induced current is to the left. $\vec{F}$ opposes the motion of the wire, as it must by Lenz's law. The magnitude of the force is $F = ILB = B^2L^2v/R$.

(b) Apply $\Sigma\vec{F} = m\vec{a}$ to the wire. Take $+x$ to be toward the right and let the origin be at the location of the wire at $t = 0$, so $x_0 = 0$.

$\Sigma F_x = ma_x$ says $-F = ma_x$

$$a_x = -\frac{F}{m} = -\frac{B^2L^2v}{mR}$$

Use this expression to solve for $v(t)$:

$$a_x = \frac{dv}{dt} = -\frac{B^2L^2v}{mR} \text{ and } \frac{dv}{v} = -\frac{B^2L^2}{mR}dt$$

$$\int_{v_0}^{v} \frac{dv'}{v'} = -\frac{B^2L^2}{mR}\int_0^t dt'$$

$$\ln(v) - \ln(v_0) = -\frac{B^2L^2t}{mR}$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2L^2t}{mR} \text{ and } v = v_0e^{-B^2L^2t/mR}$$

Note: At $t = 0, v = v_0$ and $v \to 0$ when $t \to \infty$

Now solve for $x(t)$:

$$v = \frac{dx}{dt} = v_0e^{-B^2L^2t/mR} \text{ so } dx = v_0e^{-B^2L^2t/mR}dt$$

$$\int_0^x dx' = \int_0^t v_0e^{-B^2L^2t/mR}dt'$$

$$x = v_0\left(-\frac{mR}{B^2L^2}\right)\left[e^{-B^2L^2t'/mR}\right]_0^t = \frac{mRv_0}{B^2L^2}(1 - e^{-B^2L^2t/mR})$$

Comes to rest implies $v = 0$. This happens when $t \to \infty$.

$t \to \infty$ gives $x = \dfrac{mRv_0}{B^2L^2}$. Thus this is the distance the wire travels before coming to rest.

EVALUATE: The motion of the slide wire causes an induced emf and current. The magnetic force on the induced current opposes the motion of the wire and eventually brings it to rest. The force and acceleration depend on v and are constant. If the acceleration were constant, not changing from its initial value of $a_x = -B^2L^2v_0/mR$, then the stopping distance would be $x = -v_0^2/2a_x = mRv_0/2B^2L^2$. The actual stopping distance is twice this.

30.15. **IDENTIFY:** Use the definition of inductance and the geometry of a solenoid to derive its self-inductace.

SET UP: The magnetic field inside a solenoid is $B = \mu_0 \frac{N}{l} i$, and the definition of self-inductance is $L = \frac{N\Phi_B}{i}$.

EXECUTE: **(a)** $B = \mu_0 \frac{N}{l} i$, $L = \frac{N\Phi_B}{i}$, and $\Phi_B = \frac{\mu_0 N A i}{l}$. Combining these expressions gives

$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 A}{l}$.

(b) $L = \frac{\mu_0 N^2 A}{l}$. $A = \pi r^2 = \pi(0.0750 \times 10^{-2}\ \text{m})^2 = 1.767 \times 10^{-6}\ \text{m}^2$.

$$L = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(50)^2(1.767 \times 10^{-6}\ \text{m}^2)}{5.00 \times 10^{-2}\ \text{m}} = 1.11 \times 10^{-7}\ \text{H} = 0.111\ \mu\text{H}.$$

EVALUATE: This is a physically reasonable value for self-inductance.

30.16. **IDENTIFY** and **SET UP:** The stored energy is $U = \frac{1}{2}LI^2$. The rate at which thermal energy is developed is $P = I^2 R$.

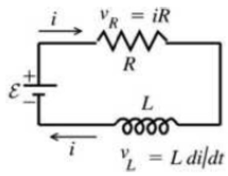
EXECUTE: **(a)** $U = \frac{1}{2}LI^2 = \frac{1}{2}(12.0\ \text{H})(0.300\ \text{A})^2 = 0.540\ \text{J}$

(b) $P = I^2 R = (0.300\ \text{A})^2(180\ \Omega) = 16.2\ \text{W} = 16.2\ \text{J/s}$

EVALUATE: **(c)** No. If I is constant then the stored energy U is constant. The energy being consumed by the resistance of the inductor comes from the emf source that maintains the current; it does not come from the energy stored in the inductor.

30.23. **IDENTIFY:** Apply Kirchhoff's loop rule to the circuit. $i(t)$ is given by Eq. (30.14).

SET UP: The circuit is sketched in Figure 30.23.



$\frac{di}{dt}$ is positive as the current increases from its initial value of zero.

Figure 30.23

EXECUTE: $\mathcal{E} - v_R - v_L = 0$

$\mathcal{E} - iR - L\frac{di}{dt} = 0$ so $i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t})$

(a) Initially $(t = 0)$, $i = 0$ so $\mathcal{E} - L\frac{di}{dt} = 0$

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} = \frac{6.00\ \text{V}}{2.50\ \text{H}} = 2.40\ \text{A/s}$$

(b) $\mathcal{E} - iR - L\frac{di}{dt} = 0$ (Use this equation rather than Eq. (30.15) since i rather than t is given.)

Thus $\frac{di}{dt} = \frac{\mathcal{E} - iR}{L} = \frac{6.00\ \text{V} - (0.500\ \text{A})(8.00\ \Omega)}{2.50\ \text{H}} = 0.800\ \text{A/s}$

(c) $i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t}) = \left(\frac{6.00\ \text{V}}{8.00\ \Omega}\right)(1 - e^{-(8.00\ \Omega/2.50\ \text{H})(0.250\ \text{s})}) = 0.750\ \text{A}(1 - e^{-0.800}) = 0.413\ \text{A}$

(d) Final steady state means $t \to \infty$ and $\frac{di}{dt} \to 0$, so $\mathcal{E} - iR = 0$.

$$i = \frac{\mathcal{E}}{R} = \frac{6.00\ \text{V}}{8.00\ \Omega} = 0.750\ \text{A}$$

EVALUATE: Our results agree with Figure 30.12 in the textbook. The current is initially zero and increases to its final value of $\mathcal{E}/R$. The slope of the current in the figure, which is di/dt, decreases with t.

30.43. **IDENTIFY and SET UP:** The emf $\mathcal{E}_2$ in solenoid 2 produced by changing current i_1 in solenoid 1 is given by $\mathcal{E}_2 = M\left|\frac{di_1}{dt}\right|$. The mutual inductance of two solenoids is derived in Example 30.1. For the two solenoids in this problem $M = \frac{\mu_0 A N_1 N_2}{l}$, where A is the cross-sectional area of the inner solenoid and l is the length of the outer solenoid. Let the outer solenoid be solenoid 1.

EXECUTE: **(a)** $M = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\pi(6.00\times10^{-4}\ \text{m})^2(6750)(15)}{0.500\ \text{m}} = 2.88\times10^{-7}\ \text{H} = 0.288\ \mu\text{H}.$

(b) $\mathcal{E}_2 = M\left|\frac{di_1}{dt}\right| = (2.88\times10^{-7}\ \text{H})(49.2\ \text{A/s}) = 1.42\times10^{-5}\ \text{V}.$

EVALUATE: If current in the inner solenoid changed at 49.2 A/s, the emf induced in the outer solenoid would be 1.42×10^{-5} V.

30.47. **IDENTIFY:** Set $U_B = K$, where $K = \frac{1}{2}mv^2$.

SET UP: The energy density in the magnetic field is $u_B = B^2/2\mu_0$. Consider volume $V = 1\ \text{m}^3$ of sunspot material.

EXECUTE: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times10^4\ \text{J/m}^3$. The total energy stored in volume V of the sunspot is $U_B = u_B V$. The mass of the material in volume V of the sunspot is $m = \rho V$. $K = U_B$ so $\frac{1}{2}mv^2 = U_B$. $\frac{1}{2}\rho V v^2 = u_B V$. The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times10^4$ m/s.

EVALUATE: The speed we calculated is about 30 times smaller than the escape speed.

30.48. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: The energy stored is $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}.$

(b) $d\Phi_B = B\,dA = \frac{\mu_0 i}{2\pi r} l\,dr.$

(c) $\Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 i l}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i l}{2\pi}\ln(b/a).$

(d) $L = \frac{N\Phi_B}{i} = l\frac{\mu_0}{2\pi}\ln(b/a).$

(e) $U = \frac{1}{2}Li^2 = \frac{1}{2}l\frac{\mu_0}{2\pi}\ln(b/a)i^2 = \frac{\mu_0 l i^2}{4\pi}\ln(b/a).$

EVALUATE: The magnetic field between the conductors is due only to the current in the inner conductor.

30.51. **IDENTIFY:** $U = \frac{1}{2}LI^2$. The self-inductance of a solenoid is found in Exercise 30.15 to be $L = \frac{\mu_0 A N^2}{l}$.

SET UP: The length l of the solenoid is the number of turns divided by the turns per unit length.

EXECUTE: **(a)** $L = \frac{2U}{I^2} = \frac{2(10.0\ \text{J})}{(2.00\ \text{A})^2} = 5.00\ \text{H}.$

(b) $L = \frac{\mu_0 A N^2}{l}$. If α is the number of turns per unit length, then $N = \alpha l$ and $L = \mu_0 A \alpha^2 l$. For this coil $\alpha = 10\ \text{coils/mm} = 10\times10^3\ \text{coils/m}$. Solving for l gives

$l = \frac{L}{\mu_0 A\alpha^2} = \frac{5.00\ \text{H}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.0200\ \text{m})^2(10\times10^3\ \text{coils/m})^2} = 31.7\ \text{m}$. This is not a practical length for laboratory use.

EVALUATE: The number of turns is $N = (31.7\ \text{m})(10\times10^3\ \text{coils/m}) = 3.17\times10^5$ turns. The length of wire in the solenoid is the circumference C of one turn times the number of turns.
$C = \pi d = \pi(4.00\times10^{-2}\ \text{m}) = 0.126\ \text{m}$. The length of wire is $(0.126\ \text{m})(3.17\times10^5) = 4.0\times10^4\ \text{m} = 40\ \text{km}.$

30.71. **IDENTIFY and SET UP:** The circuit is sketched in Figure 30.71a. Apply the loop rule. Just after S_1 is closed, $i = 0$. After a long time i has reached its final value and $di/dt = 0$. The voltage across a resistor depends on i and the voltage across an inductor depends on di/dt.

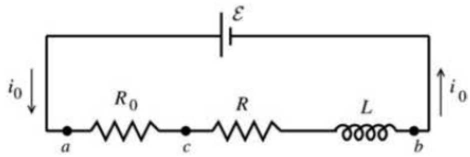


Figure 30.71a

EXECUTE: **(a)** At time $t = 0$, $i_0 = 0$ so $v_{ac} = i_0 R_0 = 0$. By the loop rule $\mathcal{E} - v_{ac} - v_{cb} = 0$ so $v_{cb} = \mathcal{E} - v_{ac} = \mathcal{E} = 36.0\text{ V}$. ($i_0 R = 0$ so this potential difference of 36.0 V is across the inductor and is an induced emf produced by the changing current.)

(b) After a long time $\frac{di_0}{dt} \to 0$ so the potential $-L\frac{di_0}{dt}$ across the inductor becomes zero. The loop rule gives $\mathcal{E} - i_0(R_0 + R) = 0$.

$$i_0 = \frac{\mathcal{E}}{R_0 + R} = \frac{36.0\text{ V}}{50.0\ \Omega + 150\ \Omega} = 0.180\text{ A}$$

$$v_{ac} = i_0 R_0 = (0.180\text{ A})(50.0\ \Omega) = 9.0\text{ V}$$

Thus $v_{cb} = i_0 R + L\frac{di_0}{dt} = (0.180\text{ A})(150\ \Omega) + 0 = 27.0\text{ V}$ (Note that $v_{ac} + v_{cb} = \mathcal{E}$.)

(c) $\mathcal{E} - v_{ac} - v_{cb} = 0$

$$\mathcal{E} - iR_0 - iR - L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = \mathcal{E} - i(R_0 + R) \text{ and } \left(\frac{L}{R + R_0}\right)\frac{di}{dt} = -i + \frac{\mathcal{E}}{R + R_0}$$

$$\frac{di}{-i + \mathcal{E}/(R + R_0)} = \left(\frac{R + R_0}{L}\right)dt$$

Integrate from $t = 0$, when $i = 0$, to t, when $i = i_0$:

$$\int_0^{i_0} \frac{di}{-i + \mathcal{E}/(R + R_0)} = \frac{R + R_0}{L}\int_0^t dt = -\ln\left[-i + \frac{\mathcal{E}}{R + R_0}\right]_0^{i_0} = \left(\frac{R + R_0}{L}\right)t, \text{ so}$$

$$\ln\left(-i_0 + \frac{\mathcal{E}}{R + R_0}\right) - \ln\left(\frac{\mathcal{E}}{R + R_0}\right) = -\left(\frac{R + R_0}{L}\right)t$$

$$\ln\left(\frac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)}\right) = -\left(\frac{R + R_0}{L}\right)t$$

Taking exponentials of both sides gives $\frac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)} = e^{-(R+R_0)t/L}$ and $i_0 = \frac{\mathcal{E}}{R + R_0}(1 - e^{-(R+R_0)t/L}$

Substituting in the numerical values gives $i_0 = \frac{36.0\text{ V}}{50\ \Omega + 150\ \Omega}(1 - e^{-(200\ \Omega/4.00\text{ H})t}) = (0.180\text{ A})(1 - e^{-t/0.0}$

At $t \to 0, i_0 = (0.180\text{ A})(1 - 1) = 0$ (agrees with part (a)). At $t \to \infty, i_0 = (0.180\text{ A})(1 - 0) = 0.180\text{ A}$ (agr with part (b)).

$$v_{ac} = i_0 R_0 = \frac{\mathcal{E}R_0}{R + R_0}(1 - e^{-(R+R_0)t/L}) = 9.0\text{ V}(1 - e^{-t/0.020\text{ s}})$$

$$v_{cb} = \mathcal{E} - v_{ac} = 36.0\text{ V} - 9.0\text{ V}(1 - e^{-t/0.020\text{ s}}) = 9.0\text{ V}(3.00 + e^{-t/0.020\text{ s}})$$

At $t \to 0$, $v_{ac} = 0$, $v_{cb} = 36.0$ V (agrees with part (a)). At $t \to \infty$, $v_{ac} = 9.0$ V, $v_{cb} = 27.0$ V (agrees with part (b)). The graphs are given in Figure 30.71b.

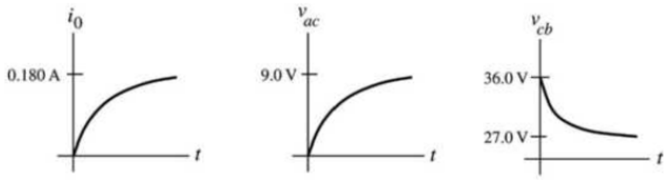


Figure 30.71b

EVALUATE: The expression for $i(t)$ we derived becomes Eq. (30.14) if the two resistors R_0 and R in series are replaced by a single equivalent resistance $R_0 + R$.

30.76. **IDENTIFY:** Follow the steps specified in the problem.
SET UP: The current in an inductor does not change abruptly.
EXECUTE: **(a)** Using Kirchhoff's loop rule on the left and right branches:

Left: $\mathcal{E} - (i_1 + i_2)R - L\dfrac{di_1}{dt} = 0 \Rightarrow R(i_1 + i_2) + L\dfrac{di_1}{dt} = \mathcal{E}$.

Right: $\mathcal{E} - (i_1 + i_2)R - \dfrac{q_2}{C} = 0 \Rightarrow R(i_1 + i_2) + \dfrac{q_2}{C} = \mathcal{E}$.

(b) Initially, with the switch just closed, $i_1 = 0, i_2 = \dfrac{\mathcal{E}}{R}$ and $q_2 = 0$.

(c) The substitution of the solutions into the circuit equations to show that they satisfy the equations is a somewhat tedious exercise but straightforward exercise. We will show that the initial conditions are satisfied: At $t = 0$, $q_2 = \dfrac{\mathcal{E}}{\omega R} e^{-\beta t}\sin(\omega t) = \dfrac{\mathcal{E}}{\omega R}\sin(0) = 0$.

$i_1(t) = \dfrac{\mathcal{E}}{R}(1 - e^{-\beta t}[(2\omega RC)^{-1}\sin(\omega t) + \cos(\omega t)] \Rightarrow i_1(0) = \dfrac{\mathcal{E}}{R}(1 - [\cos(0)]) = 0$.

(d) When does i_2 first equal zero? $\omega = \sqrt{\dfrac{1}{LC} - \dfrac{1}{(2RC)^2}} = 625$ rad/s.

$i_2(t) = 0 = \dfrac{\mathcal{E}}{R} e^{-\beta t}[-(2\omega RC)^{-1}\sin(\omega t) + \cos(\omega t)] \Rightarrow -(2\omega RC)^{-1}\tan(\omega t) + 1 = 0$ and

$\tan(\omega t) = +2\omega RC = +2(625 \text{ rad/s})(400\ \Omega)(2.00 \times 10^{-6}\text{ F}) = +1.00$.

$\omega t = \arctan(+1.00) = +0.785 \Rightarrow t = \dfrac{0.785}{625 \text{ rad/s}} = 1.256 \times 10^{-3}\text{s}$.

EVALUATE: As $t \to \infty$, $i_1 \to \mathcal{E}/R$, $q_2 \to 0$ and $i_2 \to 0$.