Solutions to Physics 9C-C MT#1 (2021)

1-(a) The force on Q at Point B is the sum of the forces exerted by q_1 and q_2 on Q.

$$\vec{F}_{on\,Q,\,at\,B} = \vec{F}_{q_1\,on\,Q,\,at\,B} + \vec{F}_{q_2\,on\,Q,\,at\,B}$$

$$= \frac{K q_1 \cdot Q}{r_{1B}^2}\hat{r}_{1B} + \frac{K q_2 Q}{r_{2B}^2}\hat{r}_{2B}$$

$$= \frac{K \cdot q_1 \cdot Q \cdot \vec{r}_{1B}}{r_{1B}^3} + \frac{K q_2 Q \cdot \vec{r}_{2B}}{r_{2B}^3}$$

$\vec{r}_{1B} = 0.04\,m\,\hat{i} + 0.03\,m\,\hat{j} = 0.05\,m\,\hat{r}_{1B}$;

$\vec{r}_{2B} = -0.04\,m\,\hat{i} + 0.03\,m\,\hat{j} = 0.05\,m\,\hat{r}_{2B}$;

$$\therefore \vec{F}_{on\,Q} = K q_1 Q \cdot \frac{1}{r_{1B}^3}\left(\vec{r}_{1B} - \vec{r}_{2B}\right) \quad (q_1 = -q_2)$$

$$= K q_1 Q \cdot \frac{1}{r_{1B}^3}\left(0.08\,m\,\hat{i}\right)$$

$$= 9.0\times10^{+9}\,N\cdot m^2/c^2 \cdot \left(10^{-9}c\right)\left(2\times10^{-9}c\right) \cdot \frac{0.08\,m}{(0.05\,m)^3}\hat{i}$$

$\therefore F_{\text{on }Q, \text{ at } B; x} = 1.15 \times 10^{-5}\ N$

$F_{\text{on }Q, \text{ at } B; y} = 0$

1-(4) The potential difference between A & B is the sum of the differences from Q_1 and Q_2.

Since $|\vec{V}_{1A}| = |\vec{V}_{2A}| = 0.04\,m$,
$|\vec{V}_{1B}| = |\vec{V}_{2B}| = 0.05\,m$,

$$V_A - V_B = (V_A - V_B)_{Q_1} + (V_A - V_B)_{Q_2}$$

$$= \left(V_A(Q_1) + V_A(Q_2)\right) - \left(V_B(Q_1) + V_B(Q_2)\right)$$

$$= \frac{K}{|\vec{V}_{1A}|}(Q_1 + Q_2) - \frac{K}{|\vec{V}_{1B}|}(Q_1 + Q_2)$$

$= 0$ since $Q_1 = -Q_2$.

Or

$$(V_A - V_B)_{Q_1} + (V_A - V_B)_{Q_2} = KQ_1\left[\left(\frac{1}{|\vec{V}_{1A}|} - \frac{1}{|\vec{V}_{1B}|}\right) - \left(\frac{1}{|\vec{V}_{2A}|} - \frac{1}{|\vec{V}_{2B}|}\right)\right]$$

1-(c) In this case,

$$|\vec{V}_{1C}| = V_{1C} = 0.04\,m$$

$$|\vec{V}_{1D}| = V_{1D} = 0.12\,m$$

$$|\vec{V}_{2C}| = V_{2C} = 0.12\,m = V_{1D}$$

$$|\vec{V}_{2D}| = V_{2D} = 0.04\,m = V_{1C}$$

$$\therefore U_C(Q) - U_D(Q)$$

$$= \left(\frac{Kq_1Q}{V_{1C}} + \frac{Kq_2Q}{V_{2C}}\right) - \left(\frac{Kq_1Q}{V_{1D}} + \frac{Kq_2Q}{V_{2D}}\right)$$

$$= Kq_1Q\left[\left(\frac{1}{V_{1C}} - \frac{1}{V_{2C}}\right) - \left(\frac{1}{V_{1D}} - \frac{1}{V_{2D}}\right)\right]$$

$$= 2Kq_1Q\left(\frac{1}{V_{1C}} - \frac{1}{V_{2C}}\right)$$

$$= 6\times10^{-7}\,J$$

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2-(a) Let the positive x direction pointing to the right.

$$\vec{E}_A = \vec{E}_A(\sigma) + \vec{E}_A(2\sigma)$$

$$= \frac{\sigma}{2\epsilon_0}(-\hat{i}) + \frac{2\sigma}{2\epsilon_0}(-\hat{i})$$

$$= \frac{3\sigma}{2\epsilon_0}(-\hat{i})$$

$$\vec{E}_B = \vec{E}_B(\sigma) + \vec{E}_B(2\sigma)$$

$$= \frac{\sigma}{2\epsilon_0}(+\hat{i}) + \frac{2\sigma}{2\epsilon_0}(-\hat{i})$$

$$= \frac{\sigma}{2\epsilon_0}(-\hat{i})$$

$$\vec{E}_C = \vec{E}_C(\sigma) + \vec{E}_C(2\sigma)$$

$$= \frac{\sigma}{2\epsilon_0}(+\hat{i}) + \frac{2\sigma}{2\epsilon_0}(+\hat{i})$$

$$= \frac{3\sigma}{2\epsilon_0}(+\hat{i})$$

2-(4) By the definition of the potential difference

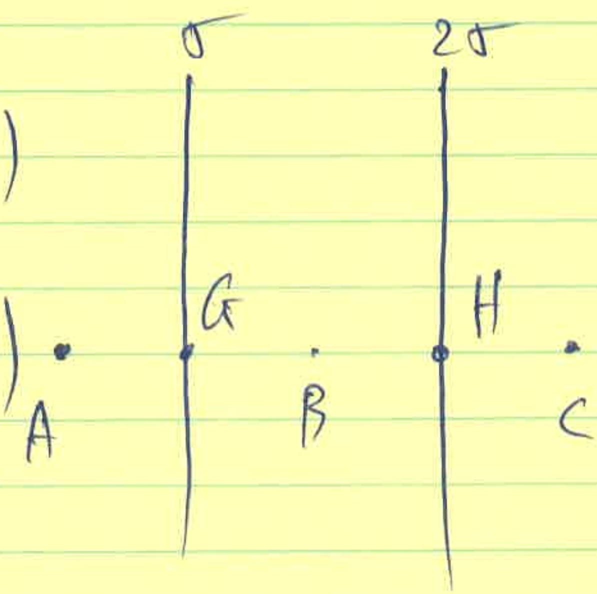
$$V_A - V_c = \int_A^c \vec{E}\cdot d\vec{l}$$

$$= \int_A^G \vec{E}_A\cdot d\vec{l} + \int_G^H \vec{E}_B\cdot d\vec{l} + \int_H^c \vec{E}_c\cdot d\vec{l}$$

$$= \frac{3\sigma}{2\epsilon_0}(-\hat{i})\,h\cdot(+\hat{i})$$

$$+ \frac{\sigma}{2\epsilon_0}(-\hat{i})\cdot d\cdot(+\hat{i})$$

$$+ \frac{3\sigma}{2\epsilon_0}(+\hat{i})\cdot h\cdot(+\hat{i})$$

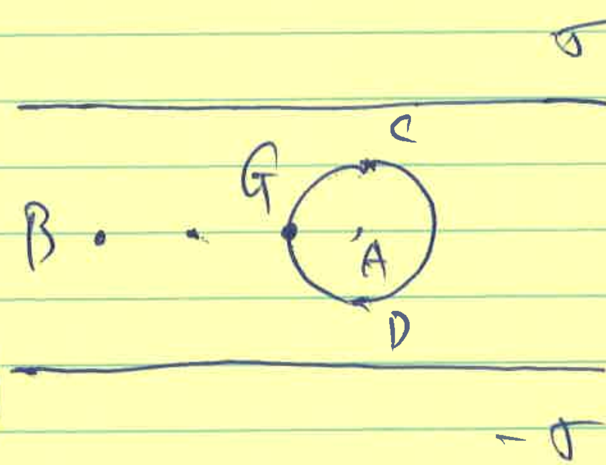


$$= -\frac{\sigma d}{2\epsilon_0}$$

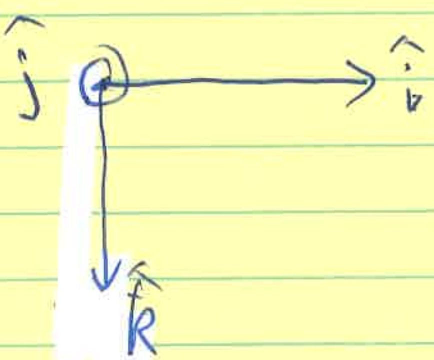
$$\therefore\ V_A - V_c = -\frac{\sigma d}{2\epsilon_0}$$

3-(a) The electric field at B is the sum of the electric fields produced by all three charge distribution

$$\vec{E}_B = \vec{E}_B(\sigma) + \vec{E}_B(-\sigma) + \vec{E}_B(\lambda)$$



$$= \frac{\sigma}{2\epsilon_0}\hat{k} + \frac{(-\sigma)}{2\epsilon_0}(-\hat{k}) + \frac{2k\lambda}{3R}(-\hat{i})$$



$$= \frac{\sigma}{\epsilon_0}\hat{k} + \frac{2k\lambda}{3R}(-\hat{i})$$

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3-(b) By definition,

$$V_A - V_B = \int_A^B \vec{E}\cdot d\vec{l} = \int_A^G \vec{E}\cdot d\vec{l} + \int_G^B \vec{E}\cdot d\vec{l}$$

Since the straight path from A to B is perpendicular to the fields produced by σ and $-\sigma$, these fields do not contribute to the integral.

Since the field inside the cylindrical shell produced by the shell is zero, the first term vanishes.

From a to B, the field produced by the shell is

$$\vec{E}(\vec{\rho}) = \frac{2k\lambda}{\rho}\hat{\rho}$$

and $d\vec{l} = \hat{\rho}\, d\rho$,

$$\therefore V_A - V_B = \int_a^B \vec{E}(\vec{\rho})\cdot d\vec{l} = 2k\lambda \ln\left(\frac{3R}{R}\right)$$

$$= 2k\lambda \ln 3$$

3-c(i) By definition,

$$V_C - V_D = \int_C^D \vec{E} \cdot d\vec{l}$$

$$= \int_C^D \left(\vec{E}(\sigma) + \vec{E}(-\sigma) + \vec{E}(x) \right) \cdot d\vec{l}$$

But $\vec{E}(x)$ inside the shell is zero, And

$$\vec{E}(\sigma) + \vec{E}(-\sigma) = \frac{\sigma}{\epsilon_0} \hat{k}$$

$$d\vec{l} = \hat{k} \cdot dz$$

$$\therefore V_C - V_D = \int_C^D \left(\frac{\sigma}{\epsilon_0} \hat{k} \right) \cdot \left(\hat{k} \cdot dz \right) = \frac{\sigma \cdot 2R}{\epsilon_0}$$

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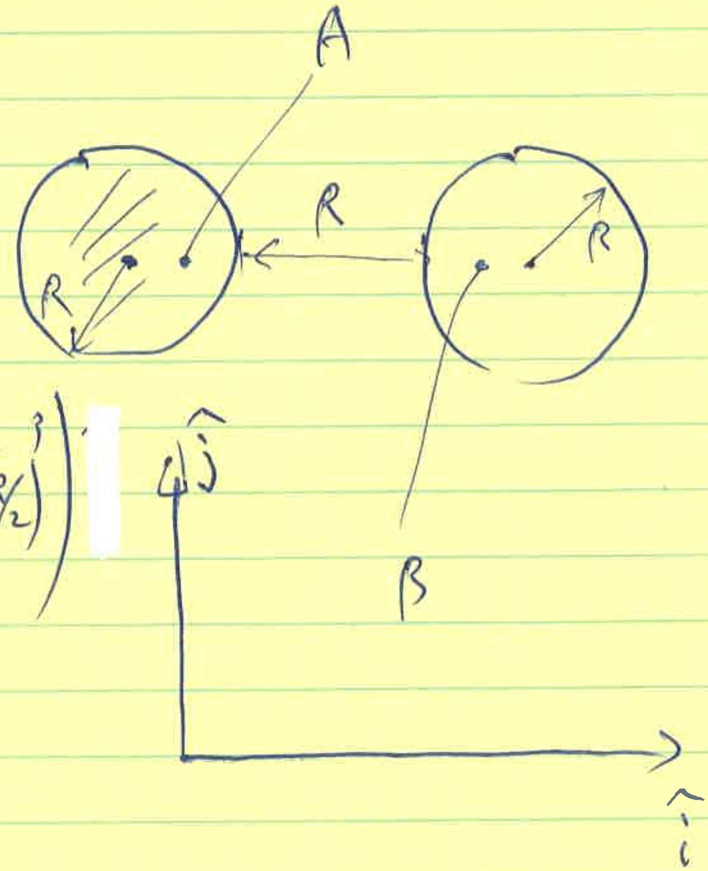
4-(a) The total electric field at A is the sum of the electric fields produced by two charge distributions at A



$$\vec{E}_A = \vec{E}_A(+Q)$$

$$+ \vec{E}_A(-Q)$$

$$= \frac{K}{(R/2)^2}\left(\frac{Q}{\frac{4\pi}{3}R^3}\right)\left(\frac{4\pi}{3}(R/2)^3\right) \cdot \hat{i}$$

$$+ \frac{K(-Q)}{\left(\frac{5}{2}R\right)^2}\cdot\left(-\hat{i}\right)$$

$$= \hat{i}\frac{KQ}{2R^2} + \frac{KQ}{R^2}(+\hat{i})\cdot\frac{4}{25}$$

$$= \frac{KQ}{R^2}\cdot\left(\frac{33}{50}\right)\left(+\hat{i}\right)$$

4-(4) electric field at $\tilde{B}$ is again the sum of the electric fields produced by the two charge distribution.

Since the negative charge $-Q$ on the shell does not produce a field inside itself, only the field produced by the solid sphere with $+Q$ is there at B.

$$\vec{E}_B = \vec{E}_B(+Q) + \vec{E}_B(-Q)$$

$$= \vec{E}_B(+Q)$$

$$= \frac{kQ}{\left(\frac{5R}{2}\right)^2}(+\hat{i})$$

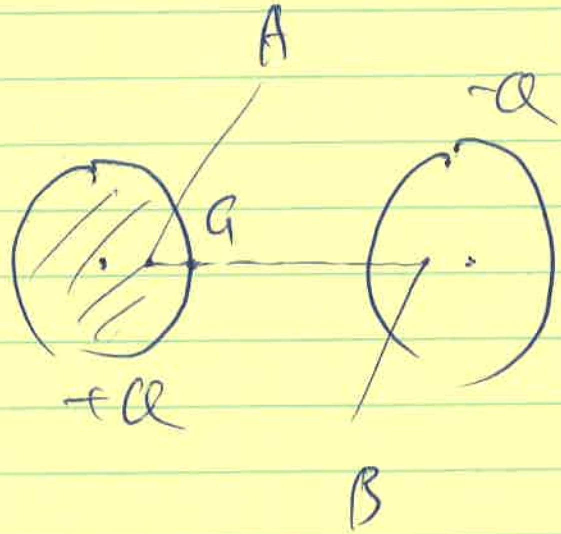
$$= \frac{kQ}{R^2} \cdot \frac{4}{25} \cdot \hat{i}$$

4-(c) In this case

$$\left(V_A - V_B\right)_{+Q} = \int_A^B \vec{E}(+Q)\cdot d\vec{l}$$

$$= \int_A^G \vec{E}(+Q)\cdot d\vec{l}$$

$$+ \int_G^B \vec{E}(+Q)\cdot d\vec{l}$$



Between A & G,

$$\vec{E}(+Q) = \frac{KQ}{R^3} r\hat{r}$$

Between G and B

$$\vec{E}(+Q) = \frac{KQ}{r^2}\hat{r}$$

$$\therefore \int_A^G \vec{E}(+Q)\cdot d\vec{l} = \frac{KQ}{R^3}\left(\frac{R^2}{2} - \frac{1}{2}\left(R/2\right)^2\right)$$

$$\int_G^B \vec{E}(+Q)\cdot d\vec{l} = KQ\left(\frac{1}{R} - \frac{1}{5R/2}\right)$$

$$\therefore \left(V_A - V_B\right)_{+Q} = \frac{kQ}{R}\left(\frac{3}{8} + \frac{3}{5}\right)$$

$$= \frac{kQ}{R}\left(\frac{39}{40}\right).$$

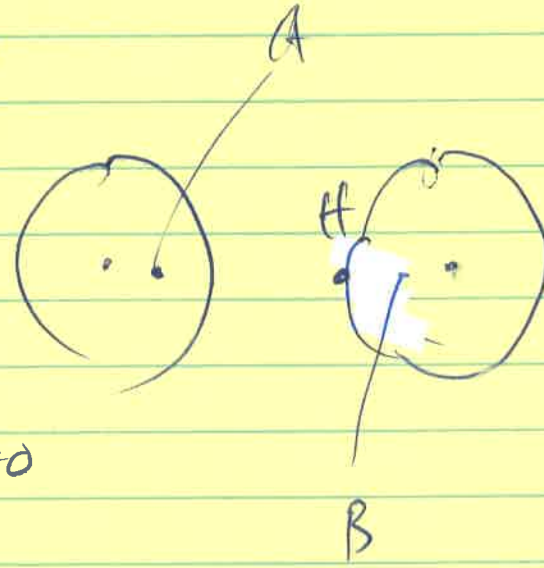
4-(d) By definition

$$\left(V_A - V_B\right)_{-Q} = \int_A^B \vec{E}(-Q)\cdot d\vec{l}$$

$$= \int_A^H \vec{E}(-Q)\cdot d\vec{l} + \int_H^B \vec{E}(-Q)\cdot d\vec{l}$$

$$= \int_A^H \vec{E}(-Q)\cdot d\vec{l}$$

$$\vec{E}(-Q)_{inside} = 0$$



$$= \frac{k(-Q)}{(5R/2)} - \frac{k(-Q)}{R} = \frac{kQ}{R}\cdot\frac{3}{5}$$

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