Physics 7B

DLM 6 Overview

Unit 4: Angular Momentum and Torque

Model/Approach: Angular Momentum Conservation Model

Conclude Act.4.3.1 **(~20 min)**

Act-4.3.2 Solidifying the Torque Construct **(~50 min)**

Learning Goals:

- Get practice determining torques, given the force and where it acts
- Develop an intuitive understanding for torques

Act-4.3.3 Rotational Inertia and Angular Momentum **(~45 min)**

Learning Goals:

- Become comfortable with the construct of rotational inertia.
- Develop an understanding of how angular speed changes are related to both angular impulse and to rotational inertia.

Act-4.3.4 Rotational Inertia and $\Delta\omega$ **(~25 min)**

Learning Goals:

- Get more practice relating angular impulses to changes in angular momentum
- Become more comfortable dealing with all of the angular motion variables

Announcements

- **Reading Assignment** - Read Summary and Review pages 101-106 and 120-124.

- Be sure to check the Physics 7B webpage periodically for new information and material. The FNTs should be available at this site as well as various announcements and all office hours

Solidifying the Torque Construct

A) DLM 05 FNTs 1, 2, 3, and 4

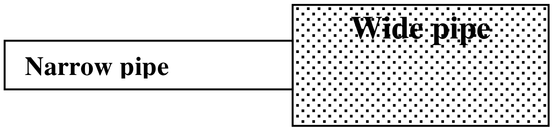
In Your Small Group

1) Compare your responses to **FNT 1**, choose a response, and put it on the board.
2) For **FNT 2**, make sure everyone in your table group can use the definition/properties of torque, (explicitly, torque = moment-arm x $F_{tangential}$) to explain why each of the forces does or does not produce a non-zero torque about the specified point.
   a) Use $\mathbf{F}_2$ to illustrate on the board the difference between the **radius** of the disk and the **moment-arm** referenced to the **point of rotation of the disk**
   b) For each force, make a drawing on the board that shows (1) the **moment arm** (a line from the pivot point to the point where the force is applied), (2) **the force vector**, (3) the **tangential component of the force**, $\mathbf{F}_{tangential}$ (remember, the tangential component is always perpendicular to the moment-arm)
3) **FNT 3**: Make sure you agree in your group the direction in space that the torque vector points for all of the forces that cause a torque.
4) **FNT 4:** Put on the board a drawing that shows how $\mathbf{F}_3$ and $\mathbf{F}_5$ can balance.

Whole Class Discussion

B) Application to the "Screwdriver"

1) Check out the torque you can apply to the pipe "screwdriver" with your fingers.



Hold the narrow pipe with the thumb and forefinger of one hand and the large pipe with the thumb and forefinger of the other hand. Apply opposite torques with each hand (turn the wide end as if you were using it to turn a screw, but turn the narrow end in the opposite direction). There are two ways you can do this:

(a) Rotate the screwdriver in opposite directions with both hands but with a net rotation of zero so neither hand wins. With which hand is it more difficult to counter the other's rotation? Why?
(b) Turn the screwdriver with the same force on each hand. Which way does the "screwdriver" turn? Why?

Make sure every group member tries this!

2) Make a circle to represent the narrow end of the pipe and show the force exerted by both your thumb and forefinger on this circle. Do the same thing for the large end of the pipe. Put these on the board. Label these as *extended force diagrams* and explain the relative magnitude of the torques exerted by each hand. Assume here that the forces exerted by each hand are the same.

   Draw a force diagram and use it to explain the observed change in the translational momentum.
3) Apply the angular impulse angular momentum relation ($\Sigma \mathbf{angJ} = \Delta \mathbf{L}$) to explain which way the "screwdriver" will turn. What must be the relation of the forces exerted by the different hands if the "screwdriver" is not going to turn?

Whole Class Discussion

Rotational Inertia and Angular Momentum

A) The Concept of Rotational Inertia

Phenomenon: Change of Momentum or Angular Momentum of a ball attached to a string and dropped from a stationary horizontal position. Focus is on the motion immediately after releasing it

Two ways to look at it

We can view the ball on the string by focusing on the ball and analyze the impulses that act to change its momentum when it is released: $\mathbf{\Sigma J = \Delta p}$

Or, we can view the ball and string as an extended object and apply and analyze the rotational impulses that act to change the ball and string's angular momentum when it is released: $\mathbf{\Sigma angJ = \Delta L}$

Instead of mass, we now have rotational inertia, I, and instead of a change in translational momentum, we have a change in angular momentum. We should get the same result for the change in the velocity of the ball in either case. Remember that for a point particle the translational and rotational momentum are related as

$$v_{tangential} = r\,\omega \text{ and } \Delta v_{tangential} = r\,\Delta\omega \quad \text{or} \quad \Delta\omega = \Delta v_{tangential} / r$$

(Follow your DL instructors demonstration of the equivalence of these two approaches.)

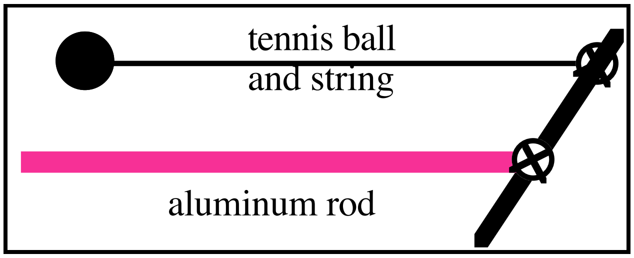
Rotational Inertia, I, depends on the mass and its distance from the rotation point.

For mass located a distance r from rotation point,

Important Result: $\mathbf{I = m\,r^2}$

This is the basic relationship for rotational inertia. Extended objects will have a numerical coefficient. For example, for a disk rotating about its center, $I_{disk} = 1/2\ mr^2$. For a rod rotating about its end, $I_{rod,\ end} = 1/3\ mL^2$ The chart on page 31 of the Course Notes lists the rotational inertia for a variety of shapes.

B) Applying the Concepts



Do and Observe

- Setup the rod and tennis ball and string so they are the same length (to center of ball).
- Hold them both horizontally and release.
- Observe the changes in their rotational velocities. They both start with zero velocity. Which has the greater initial change in rotational velocity?

Analyze and Explain

Draw *extended* force diagrams on the board for both rod and ball and note any differences.

For the moment, let's assume that the ball and rod have the same mass. Which one has the greater torque exerted upon it? Which has the greater moment of inertia? Using angular impulse and ΔL to analyze and explain why there is a greater change in Δω. That is, why does one rotate faster?

Bonus Question: The ball and rod do not really have the same mass. Nonetheless, the rod "wins" the race. Why doesn't the mass difference matter?

Another extra question: Stand two inertia wands side-by-side but the heavier end of one near the floor and the other opposite. Predict which will fall on the ground first? Now try this and explain using the same steps of analysis as before.

Rotational Inertia and Newton's 2nd Law

A) Experiencing Rotational Inertia

Each person in the group must do this!

- Hold the rotational inertia wand at one end in your outstretched arm, so that it is vertical. Keeping your arm should be straight, rigid, and stationary, rotate the rod rapidly back and forth, pivoting it at your wrist. You should simply rotate your wrist about the long axis of your arm.
- Repeat, holding the wand at the other end.

Explain the difference in the rotational motion:

In your small group develop an explanation for the difference in rotational motion for the wand when holding it at its different ends. Use the concepts of change in angular momentum and angular impulse. Make an angular momentum chart for each case.

Share: **Be ready to share your explanation**

B) Applying the Idea of Rotational Inertia

FNT 6: Put your group's response to this FNT on the board.

FNT 5. Put your group's response to this FNT on the board.

Followup: Using what you know about rotational inertia, predict the relative angular speeds of Jill and the merry-go-round. Who will rotate faster? Why?

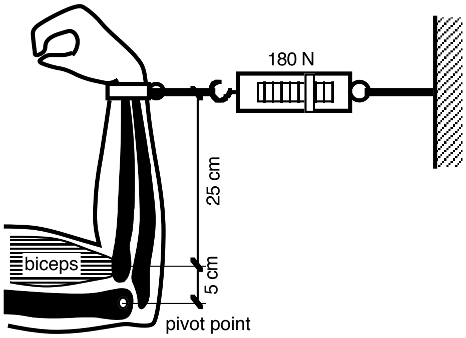
Physics 7B

Exit Handout

DLM 06

FNTs Read the Course Notes on the Angular Momentum Model, pages 26 – 37. Work hard on seeing the analogies between linear momentum phenomena and angular momentum phenomena.

1) A physical therapy patient contracts her biceps muscle, and exerts a horizontal force of 180 N on the spring shown in the figure at right. Assume the forearm rotates at the elbow.



a) Take your object to be the forearm and draw an extended force diagram of the forearm showing the rotation point and all of the forces acting on the forearm *where they actually are applied*. You won't know the magnitudes (or even the directions) of these forces until you finish the whole problem but just draw reasonable vectors for now.

b) Draw a second force diagram with the forearm as a dot.

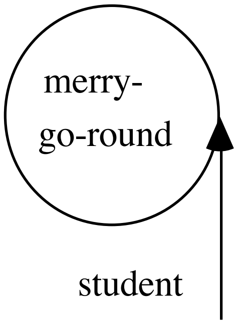
c) Use the fact that all the forces and all the torques must sum to zero (Why are $\Sigma F = 0$ and $\Sigma\tau = 0$ in this situation?) to find the horizontal force that her biceps exerts on her forearm and the force exerted by the bone of the upper arm (shown below the biceps) on the forearm (weight = 30N). **Note that there is a horizontal component to the force of the upper arm as well as a vertical one—you will need to use the model to determine the direction of the force, because it is not intuitively obvious).**

d) Explain in complete English sentences (i.e., not just equations) why the bone of the upper arm exerts such a large force on the forearm.

e) What is the advantage of having the point where the biceps is attached to the forearm so close to the elbow?

f) What is the disadvantage of having it attached so close?

2) A physics 7 student runs along a line tangent to the edge of a motionless merry-go-round and jumps on at the very outside. The merry-go-round has the shape of a uniform disk. Refer to the chart of the analogs of rotational variables to linear variables in the Course Notes to help you with this problem.



(a) Make an angular momentum chart to help you keep track of what is and is not changing. Treat the two objects, the student and merry-go-round together, as a system in which angular momentum is conserved (Hint angular momentum conservation depends on your choice of coordinate system, rotation with respect to a particular origin or pivot).

(b) Is linear momentum conserved in this interaction?

(c) When the person is running toward the merry-go-round they have kinetic energy, of course. What happens to this energy after they jump on the merry-go-round? Is this kinetic energy conserved? If not, which energy systems does it go to?

(d) Based on your answers, would you say the collision of the person with the merry-go-round is elastic or inelastic? Explain.

3) Consider what you know about rotational inertia.

(a) Explain in words (i.e., not just equations) why the vertical thickness of a solid disk does not figure in the calculation of its rotational mass about its center.

(b) A biology student notices that mammals that run fast have slender legs with muscles concentrated in the body. Give a plausible explanation for this observation.