

## Unit 4: Angular Momentum and Torque

### Model/Approach: Angular Momentum Conservation Model

#### Act-4.3.5 Conservation of $L$ on a Rotating Stool

(~50 min)

Learning Goals:

- Become more comfortable with the application of conservation of angular momentum to a multi-part closed system.
- Develop a deeper understanding of how angular speed changes are related to both angular impulse and to changes in rotational inertia.

#### Act-4.3.6 Examples of $\Delta L = 0$ FNTs 1 and 2 from DLM 6

(~40 min)

Learning Goals:

FNT 1

- Get practice determining torques, given the force and where it acts
- Get practice using  $\Delta \mathbf{p} = 0$  and  $\Delta \mathbf{L} = 0$  to infer  $\Sigma \mathbf{F} = 0$  and  $\Sigma \boldsymbol{\tau} = 0$ .
- Get practice using  $\Sigma \mathbf{F} = 0$  and  $\Sigma \boldsymbol{\tau} = 0$  simultaneously to find unknown forces.

FNT 2

- Get practice applying conservation of  $L$  to rotational collision problems
- Get practice using the relation  $L = r p_{\text{tangential}}$

#### Act-4.3.7 Torques and $\Delta L$ 's of a Bike Wheel

(~50 min)

Learning Goals:

- Develop better understanding of the vector nature of angular impulse and how it depends explicitly on applied forces
- Become more comfortable with using the vector nature of  $\boldsymbol{\tau}$  and  $\Delta \mathbf{L}$  to predict changes in motion of a bike wheel under various angular impulses

## Announcements

- **Reading Assignment** - Read Unit 5.
- Be sure to use the Physics 7B webpage periodically for new information and material. Look for the links to help you understand the bike wheel's precession.

## Conservation of L on a Rotating Stool

### A) Applying the Idea of Rotational Inertia (If you did not finish this in DL 6)

FNT 6 (from DL 5). Put your group's response to this FNT on the board.

FNT 5 (from DL 5). Put your group's response to this FNT on the board.

**Followup:** Using what you know about rotational inertia, predict the relative angular speeds of Jill and the merry-go-round. Who will rotate faster? Why?

### B) Changing the Rotational Inertia

**Phenomenon:** Change of rotational inertia while spinning on a rotating stool.

**Each person in the group must do this! This means every person in the group!**

- 1) Holding two dumbbells in close to your body, spin around on the rotating stool. Now push the dumbbells straight out to arm's-length.

Was there a net torque applied to the system of [you + rotating stool + dumbbells]? Explain whether  or not. Explain how this accounts for what you observed, by considering changes in the rotational inertia,  $I$  (Remember  $L = I\omega$ )

Make sure everyone in your group can give an explanation for the change in motion when the changes described in this part of the activity were made.

- 2) Come to a consensus and post your answer to FNT 3 from Exit Handout 6. How does this relate to what you did above in part 5?

### C) Directions of all Rotational Vectors

**Phenomenon:** Change of Angular Momentum of parts of a rotating system. Holding a spinning bike wheel on a stool.

**Each person in the group must do this! This means every person in the group!**

- 3) With the bike wheel spinning fast and the axis of rotation **vertical**, quickly turn it end-over-end **once while sitting on an initially stationary stool**.
- 4) Consider you and the stool together to be one part and the bike wheel to be a second part of one physical system. Is the torque you exert on the bike wheel an **external** or **internal** torque? What happens to the angular momentum of the entire physical system? (If you do this quickly, are frictional torques exerted by the wheel-bearing significant? How does the angular momentum of the two individual parts change?)
- 5) Make an angular momentum chart with one object being "you and the stool" and the other object being the bike wheel.
- 6) Use your momentum chart to explain and make sense of what you observe

Make sure everyone in your group can give an explanation for the change in motion observed when the changes described in this part of the activity were made.

## Examples of $\Delta L = 0$

### A. DLM 06 FNT 2

#### In Your Small Group

- 1) Discuss and come to a consensus on the angular momentum chart for physical situation in FNT 2. Put this chart on the board.
  
- 2) Put your group's consensus responses to FNT 2 parts (a) – (d) on the board.

#### Whole Class Discussion

### B. DLM 06 FNT 1

#### In Your Small Group

- 1) Discuss and come to a consensus on what the explicit implications are of the fact that the arm in **FNT 1** is not changing its translational motion or its rotational motion. Write this out on the board in two complete sentences of the form: "Because the ... is not changing, we know that...."
- 2) Put your group's consensus *extended* force diagrams from FNT 1 part (a) on the board. Then draw a force diagram with the arm represented as just a point, as in part (b) (Representing objects as points is what is generally done in force diagrams. This is done when seeking the net force. If the net torque is sought, then an extended force diagram is used for representing where on the object(s) are the forces). Make sure you have met all conditions described in question 1 above!
- 3) Put a summary on the board of how you found the forces asked for in FNT 1 part (c). Show these forces with correctly sized arrows on your two force diagrams. How did having both diagrams help find the forces?
- 4) Did you find a useful relationship that relates forces perpendicular to lengths of lever arms (only if  $\Delta L = 0$ )? (Hint: Twins Joe and Jane sit on a balanced seesaw. If Jane sits twice as far away as Joe, how much heavier is he?)
- 5) Discuss your response to FNT 1 part (d) and put your consensus response on the board.
- 6) Discuss your responses to (e) and (f) and be prepared to share them with the class.

#### Whole Class Discussion

## Torques and $\Delta\mathbf{L}$ 's of a Bike Wheel

### A) Directions of all Rotational Vectors

**Phenomenon:** Change of Angular Momentum of a bike wheel with its axis held vertically.

- 1) Rotate a bike wheel at constant speed with its axis vertical. **Demonstrate** how a (net) torque should be applied to *increase* the magnitude of the angular momentum  $\mathbf{L} = I\boldsymbol{\omega}$  of the bike wheel.
  - a. Identify the moment arm  $\mathbf{r}$  and the direction of  $\mathbf{F}_{\text{tangential}}$ .
  - b. What is the direction of the applied torque,  $\boldsymbol{\tau}$ ? What is direction of the applied impulse,  $\text{ang}\mathbf{J} = \boldsymbol{\tau}\Delta t$ ?
  - c. Draw the vectors representing  $\boldsymbol{\tau}\Delta t$ ,  $\Delta\mathbf{L}$ ,  $\mathbf{L}_i$ , and  $\mathbf{L}_f$  on the board on a sketch of the bike wheel. In your drawings, choose the viewing perspective that shows the vectors most clearly.
- 2) Repeat, but show the vectors when you apply a force that *decreases* the magnitude of  $\mathbf{L}$ .

**Make sure everyone in your group can explain how you “speed up” the wheel by applying a force. Everyone must understand the directions of all the vectors. Don’t leave this until everyone in your group gets it!**

### B) Experiencing the directional nature of angular impulse and $\Delta\mathbf{L}$

**Each person in the group must do this! This means every person in the group!**

- 3) Hold a bike wheel (**non-spinning**) with its axis horizontal straight out in front of you at shoulder height. **Arms straight and horizontal and elbows locked. QUICKLY** push one end of the axle out a small amount and simultaneously pull the other end a little closer to you, by *moving your shoulders slightly*. Determine the direction of the  $\boldsymbol{\tau}$  you exerted.
- 4) Next, get the wheel rotating at a high speed and repeat the procedure. How does the axle “**want**” to move? Let it move the way it “wants to move”. Don’t resist it. How are the directions your hands are pushed by the wheel’s axles related to the initial directions you tried to move your hands? How did  $\mathbf{L}$  “want” to change? Assuming you let it change the “way it wanted to”, what was  $\Delta\mathbf{L}$ ? Develop an explanation of this by determining the net angular impulse,  $\text{ang}\mathbf{J} = \boldsymbol{\tau}_{\text{net}}\Delta t$ .
- 5) Make a drawing on the board showing the vectors  $\mathbf{L}_i$ ,  $\Delta\mathbf{L}$  and the net impulse,  $\boldsymbol{\tau}_{\text{net}}\Delta t$ .

Make sure everyone in your group can give an explanation for this motion of the spinning wheel. You should be able to start with the horizontal forces you exert on the axle.

### C) A Bike wheel that doesn’t “fall”

- 6) Get the bike wheel spinning as fast as you can. Then, while one person holds the wheel horizontally by the “handles”, another person should securely hold the rope loop in a vertical position. Then, the person holding the handles lets go completely.
 

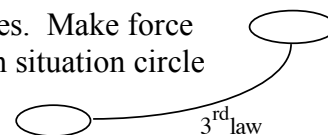
Observe and describe what happens when only the rope supports the spinning wheel.
- 7) Figure out the torque caused by the Earth’s pull acting at the center of gravity using the point where the rope is attached as the point of rotation. Draw vectors representing the impulse caused by the torque, and the initial and final angular momenta (let  $\Delta t$  be about a second). Show how this torque causes the wheel to precess (the motion you see) rather than simply “fall down.”
- 8) Make a drawing on the board showing the vectors  $\mathbf{L}_i$ ,  $\Delta\mathbf{L}$  and the net impulse,  $\boldsymbol{\tau}_{\text{net}}\Delta t$ .

**DLM 7 FNTs**

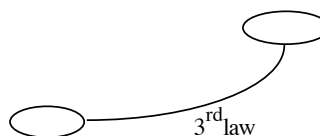
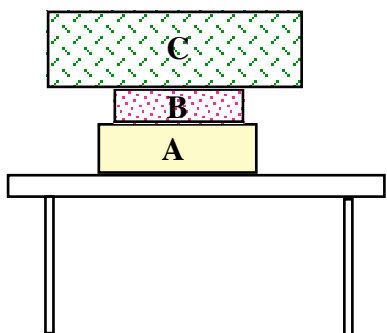
- 1) Finish the analysis of any parts of Activities 4.3.6 and 4.3.7 you did not completely finish. Your DL instructor will collect your work and grade it.

**Newton's Laws:**

- 2) Create an example for each of Newton's three laws. Describe each law in both sentence form and with a concise mathematical expression. Be careful and precise with notation in the algebraic expressions.
- 3) Review the rules for drawing force diagrams as given in the Course Notes. Make force diagrams for the following situations. Follow the rules exactly. For each situation circle the forces that are in tandem as third law pairs, as shown.



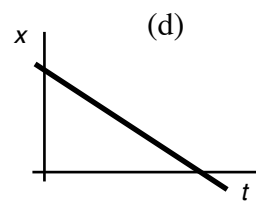
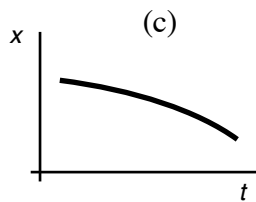
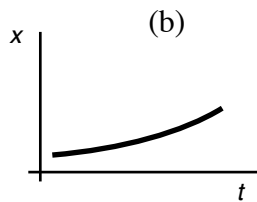
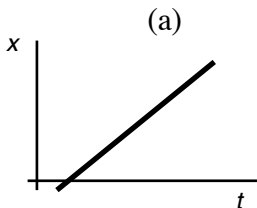
- a) Refer to the rocket example of FNT 3 in DL 2. Recreate a momentum chart and follow the instructions above. In order for the rocket to move at a constant speed does it need to have a net force on it? Explain how each of Newton's Laws can be identified in the momentum charts and force diagrams.
- b) Refer to the boat example of FNT 4 in DL 2. Recreate a momentum chart and follow the instructions above. Explain how each of Newton's Laws can be identified in the momentum charts and force diagrams.
- 4) Static example: Make a force diagram of **each** of the four objects; books A, B, and C, and the table. The table is sitting on the floor. The masses of the books are 0.8 kg, 0.2 kg, and 0.5 kg for books A, B, and C, respectively and the mass of the table is 30 kg.



- 5) Use Newton's 1<sup>st</sup> and 3<sup>rd</sup> laws to numerically determine all of the forces acting on the books and the table. Redraw your force diagrams so they are more to scale.
- 6) Identify all 3<sup>rd</sup> law pairs of forces appearing in your two force diagrams by circling the two forces and connecting as shown in the diagram above.
- 7) If the books and table are sliding across the room each at the same constant speed, how would your force diagrams change? Show precisely how it would change or explain why it would not.

**Plotting Motion**

- 8) State whether the acceleration is positive, negative, or zero for each of the position functions  $x(t)$  in the position versus time graphs below. How do you know?



- 9) For each of the following scenarios make a position vs. time graph. Directly below it draw a velocity vs. time graph, and beneath that draw acceleration vs. time.
- A dropped object as it is falling (before it hits the floor).
  - A rocket firing its engines for a certain length of time descending on Mars.
  - A racecar during the first 10 seconds after it starts from a stop.
- 10) Beneath the appropriate column of graphs from #8, write an equation that solves for the variable in question (see below). Write which model you used: Newtonian 2<sup>nd</sup> Law Model, energy interaction model, the momentum conservation model, the angular momentum conservation model, etc... If you introduce any new variables, clearly indicate what they mean.
- The velocity of a dropped object just before it hits the floor.
  - The time it takes the dropped object in (8a) to reach the floor after being dropped.
  - The change in velocity from a spacecraft firing its rockets for a certain length of time.