

Model: Linear Transport Model

AC-6.3.3 Heat flow FNT from DLM 15 (No activity sheet) (~ 15 min)

Model: Exponential Change

AC-7.1.2 Introduction to Capacitors and Capacitance (~ 25 min)

Learning Goals:

- Begin to think about non-stead-state currents
- Get an introduction to a capacitor

AC-7.1.3 Electrical Capacitor Discharging (~40 min)

Learning Goals:

- see AC 7.1.2
- Get familiar with RC circuits.

Act-7.1.4 Physics of Exponential Change DLM 15 FNTs (~ 60 min)

Learning Goals:

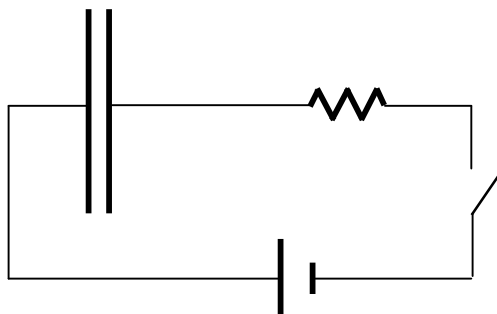
- Understand how a physical system that behaves as $dy(t)/dt = -\lambda y(t)$ leads to exponential behavior
- Practice seeing how the properties of a physical system determine the constant λ
- know that $1/\lambda$, the time constant, for a resistor-capacitor is the product RC

Announcements

- **Reading Assignment** - Finish reading Unit 7.
- Begin studying for the **final** now. Do a little bit each day by going over the FNTs, quizzes, DLM activities, and practice problems.

Introduction to Capacitors and Capacitance

Consider the two pieces of metal very close to each other, but not touching. Each plate is suddenly connected to the two ends of a battery through a resistor R (use a light bulb) as shown.



The two metal plates are called a capacitor. The metal plates can accept electrons or lose electrons.

What happens when the battery is first connected?

Usually we have thought about current in terms of positive charge, but here it is useful to focus on the electrons themselves, since they are the charge carriers in metal conductors.

- When the switch is closed (connection is made) can there be a **steady-state** current? Explain? Hint: think about the conductivity of air as opposed to the conductivity of copper wire. Does air normally conduct electricity?
- When the switch is closed (connection is made) can there be a current that lasts for a short time? Explain?
- Focusing on the two metal plates, what physically is happening to electrons immediately after the switch is closed? Are they leaving a plate? Going to a plate? What is happening to the net charge on each plate?
- What would the time dependence of the current in the resistor be like immediately after closing the switch? Make a sketch of the value of this current as a function of time. What would its maximum value be? Explain.
- Put your response to **FNT 3** on the board after discussing each part in your SG.

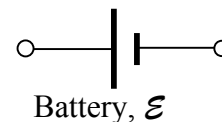
Electrical Capacitor Discharging

The phenomenon: An electrical capacitor is charged and discharged.

- a) You have a capacitor, a battery, a resistor, and a voltmeter, along with some wires to hook these elements up. You also have a holiday light bulb. Make sure you have everything. Set the voltmeter so it can read the voltage of the battery and check to see that the battery is not “dead”. Check to see that the bulb works as well. Determine the value of your resistor and record it here ____.



- b) “Charge up the capacitor” by attaching it to the battery. Then disconnect the battery and measure the voltage across the capacitor. It should be similar to the voltage of the battery. Why?



- c) Hold the ends of the bulb across the capacitor and observe what happens. Describe this in a sentence or two. Based off this observation, what can you infer from a capacitor does with electrical charge?
- d) The capacitance of a capacitor is defined to be $C = q/\Delta V$. The unit coulomb/volt is given the name farad. The capacitor you are using is extremely large. It has a capacitance of nearly 1 farad. Assuming your capacitor is 1 farad, approximately how many electrons did you put on one plate of your capacitor when you “charged it up” with the battery? (Note: one electron has a charge of 1.6×10^{-19} Coulombs)

Collect the data

Make sure to collect data so that you can do the next activity (7.1.4) and FNTs.

- e) Charge your capacitor up, attach the voltmeter across the capacitor, and then attach the resistor across the capacitor so you can measure the voltage of the capacitor as it discharges. Measure the voltage of the capacitor as a function of time as it discharges through the resistor. Take data until the voltage has reduced to below one-tenth its initial value. This is your $V(t)$ data.
- f) Plot your data on a sheet of graph paper. Fit (eyeball) a smooth curve through your data points. Remember, you want a best fit curve; the curve doesn’t actually have to go through each point. Then sketch your graph on the board.
- g) Determine the half-life of your capacitor/resistor combination from your graph. The half-life is the time taken for the initial voltage to decrease to one-half that voltage.
- h) Determine the time for the voltage to drop to $1/e$ of its original value and record it here _____. This is referred to as the **time constant** (when the voltage is equal to $1/e = 0.368$ of its original value).
- i) Now take and record measurements of **charging** your capacitor through the *same* resistor (light bulb or other): Do this by attaching the capacitor, battery, and resistor all in series (see circuit in Act. 7.1.2). Put the voltmeter across the capacitor to measure its voltage. Start with the capacitor totally discharged (use your bulb to discharge the capacitor), and take time and voltage readings as the capacitor charges up. Make a table of your readings and **calculate** a new variable, $V_{\text{fully charged}} - V(t)$. Plot this voltage as a function of time on the same graph. What can you conclude about the curves as well as the half lives (and time constants)?

Whole Class Discussion

Physics of Exponential Change DLM 15 FNTs

A) Experimentally seeing the fundamental condition that produces exponential behavior

In your small group

1) FNT 2 - using your Volume data

- a) Use one of the graphs from your group members. Determine from your graphs of $\text{Vol}(t)$ and $d\text{Vol}/dt$, what number (with units) you would have to multiply the curve of $\text{Vol}(t)$ to make it equal the curve of $d\text{Vol}/dt$. That is, determine the constant λ in the relation: $d\text{Vol}/dt = -\lambda \text{Vol}(t)$.
Fill in the following chart for several time values:

time	Volume	dVol/dt	λ

- b) What do you notice about the λ ? Describe in your own words what you did in this FNT, in the question above, and the significance of what you found. That is, answer the question, “What does all this mean?”

Whole class discussion

- 2) **Using your Voltage data** - Using the data from part e) of the last activity complete parts a) and b) that you just finished for the flow out of a standpipe.

Whole class discussion

B) Interpreting the time constant

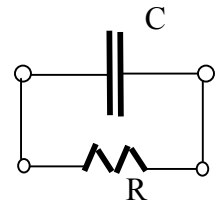
- 1) The inverse of the constant λ in $e^{-\lambda t}$ is called the time-constant; i.e., $1/\lambda = \text{time-constant}$. In electrical phenomena, it is called the “RC time-constant”.
- a) By how much has the voltage decayed from a series resistor-capacitor circuit when the time is equal to one time constant?
 - b) How many RC time constants must pass for the voltage to have decayed to less than 5 % of its original value? (use your calculator: see how much it decreases for each successive time constant)
 - c) Use the RC time constant you found in Activity 7.1.3 part (h) and the value of the resistor you found in part (a) to calculate the actual value of the capacitor you used.
- 2) a) Use a calculator to “experimentally” (don’t use a formula) determine how many years it would take to reduce the radioactivity at a waste site to less than 0.01 of its initial value, if the dominant radioactive isotope had a half-life of 50 years. Put your work on the board.
- b) Draw a graph representing this decay and label your axes. Plot a curve if the time constant were 5 years or 500 years.

Whole class discussion

FNTs

Wrap-up of Exponential Change (you derive the equation).

- 1) When a capacitor is discharging, the circuit is simply a capacitor and resistor hooked in series—end-to-end. From the definition of capacitance, the voltage across the capacitor ΔV is given by the expression: $\Delta V = q/C$. You know by definition that the current I is simply the time derivative of the charge: $I = dq/dt$.



Apply the energy-density relation from one side of the resistor to the other. Re-arrange this expression, using the substitutions suggested in the two previous sentences to get it in the form $dq/dt = -\lambda q$. What is λ equal to in terms of the other electrical constants in this situation?

Beginning of Simple Harmonic Motion (SHM)

The most general form of the equation that describes *any* object undergoing SHM is given by:

$$y(t) = A \sin\left(\frac{2\pi t}{T} + \phi\right) + B$$

A is the amplitude, T is the period of oscillation, ϕ is a constant phase factor, and B is the equilibrium value of $y(t)$, if it is not zero. We will normally let $B = 0$. The next several FNTs have to do with making sense of this relationship.

- 2) Use the general expression above, with $B = 0$, to sketch the following graphs, using the same time axis for all of them (all on the same graph). Your time axis should go from $t = 0$ to $t = 10$ s.
- I) $A = 2$, $T = 5$, $\phi = 0$ II) $A = 2$, $T = 5$, $\phi = \pi/2$ III) $A = 2$, $T = 5$, $\phi = -\pi/2$
- Explain in words how you knew how to “start” each graph where $t = 0$.
- 3) From your graph, describe in words the motion of the object described by the equation in FNT 2 for case II) at the particular times when $t = 0$ and when $t = 1.25$ s. What is its position and qualitatively describe its velocity.
- 4) a. Using your knowledge of a mass-spring developed in 7A, relate energy to amplitude in variable form. What is the object’s maximum speed in variable form?
 b. For a mass-spring system of mass m and spring constant k , which alteration will achieve a greater maximum speed; compressing a mass of $2m$ to $x_{\max} = A$, or compressing a mass $4m$ to $A = 2x_{\max}$?
- 5) An object moves with simple harmonic motion. If the amplitude and period are both doubled, the object’s maximum speed is
- a. quadrupled c. unchanged e. quartered
 b. doubled d. halved
- 6) Draw three *independent* pictures of a mass-spring system undergoing an oscillation at $t = 0$ with a phase constant, ϕ , that equals i) $\pi/4$ rad ii) $-\pi/4$ rad iii) $3\pi/4$ rad. For each case label the amplitude (each has amplitude A) and place a velocity vector on the mass to indicate its direction of motion.